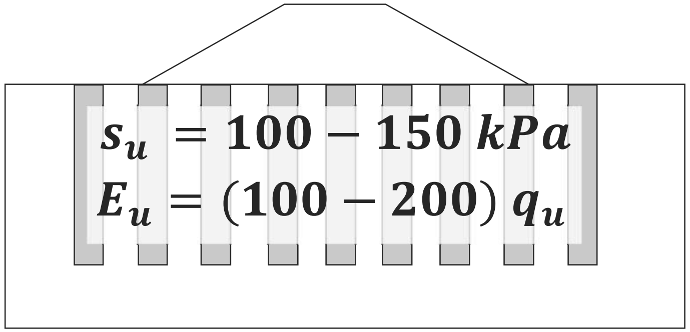
Numerical modelling of stabilised geostructures – embankments and excavations

Minna Karstunen (based on work of Urs Vogler, Sinem Bozkurt & Ayman Abed)

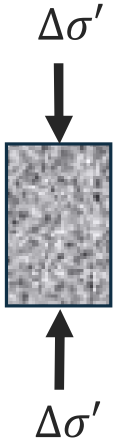
Outline

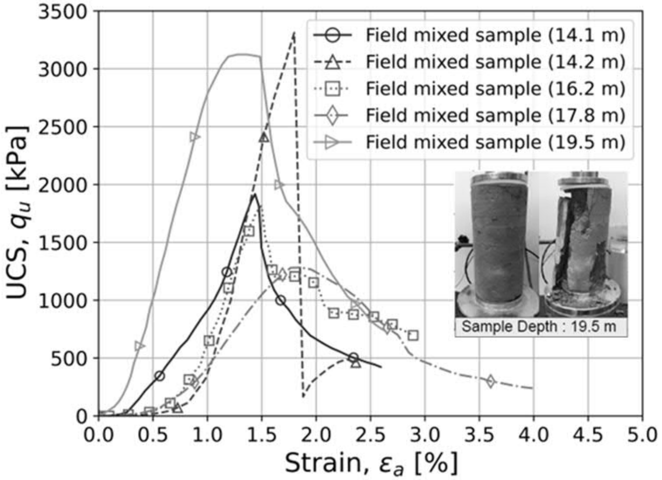
- Background & motivation
- Introduction to Volume Averaging Technique (VAT)
- VAT for embankments
- VAT for excavations
- Sensitivity analysis using VAT for embankments
- Conclusions
- Future studies

Background & Motivation



Trafikverket (2014)

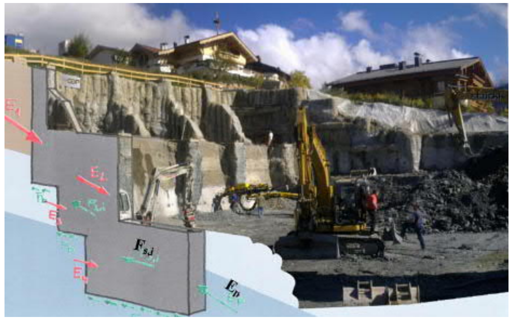




Design based on UCS strength, but in most applications key consideration is actually stiffness (settlements, horizontal movements and vibrations)

Bozkurt et al. (2023)

Topolnicki (2004)



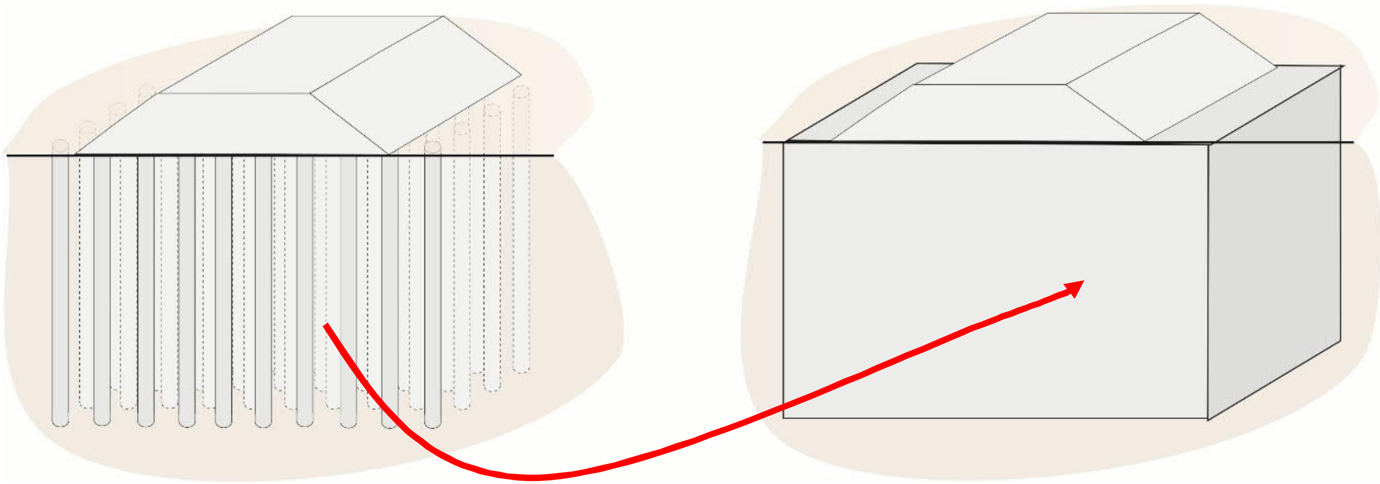
Marte et al. (2017)

Aim and objectives

Investigating time-dependent stress-strain response of deep excavations and embankments stabilised with lime-cement (LC) columns in soft clays

- Validate VAT for embankments
- Identify the interaction mechanism between ***clay & column material***
- Derive a volume averaging technique ***(VAT)*** for excavations
- Inspecting the factors having the largest impact on ***the system response*** and associated anthropogenic greenhouse gas emissions

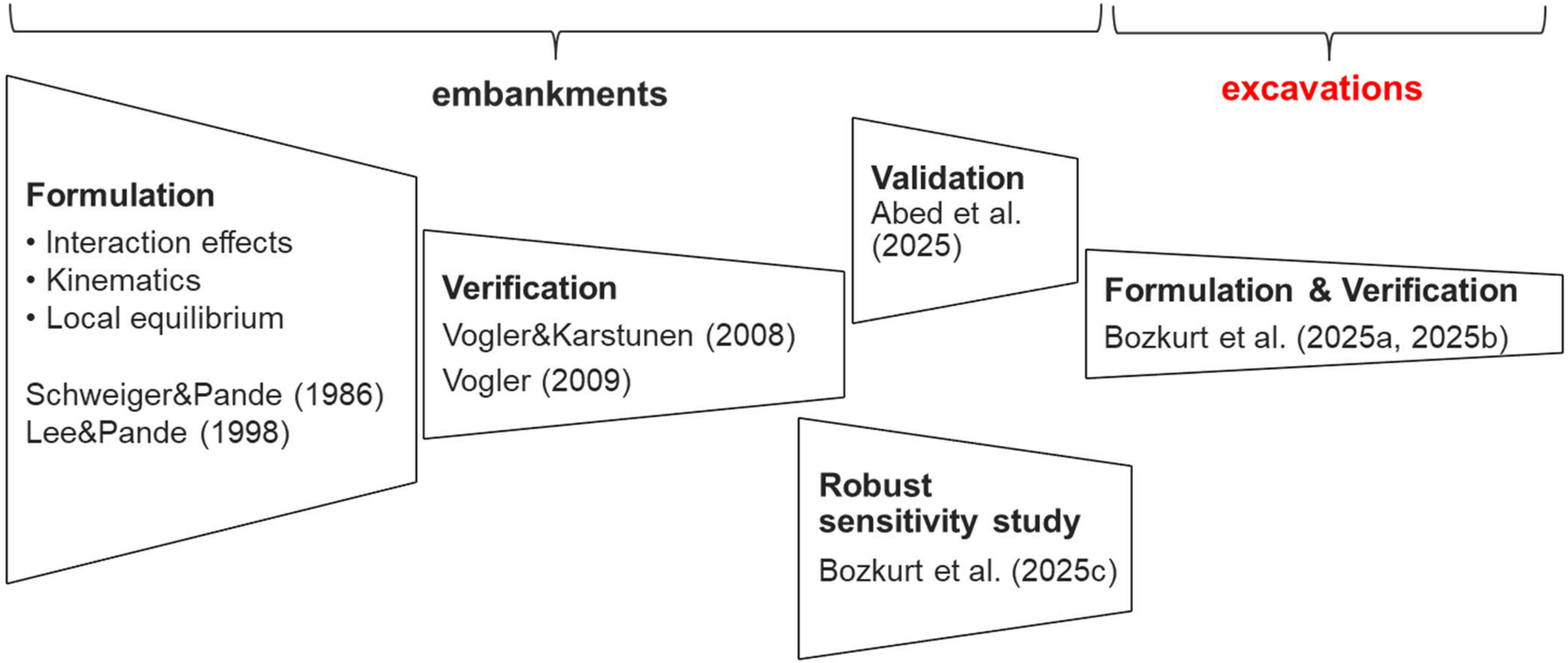
Volume Averaging Technique (VAT)



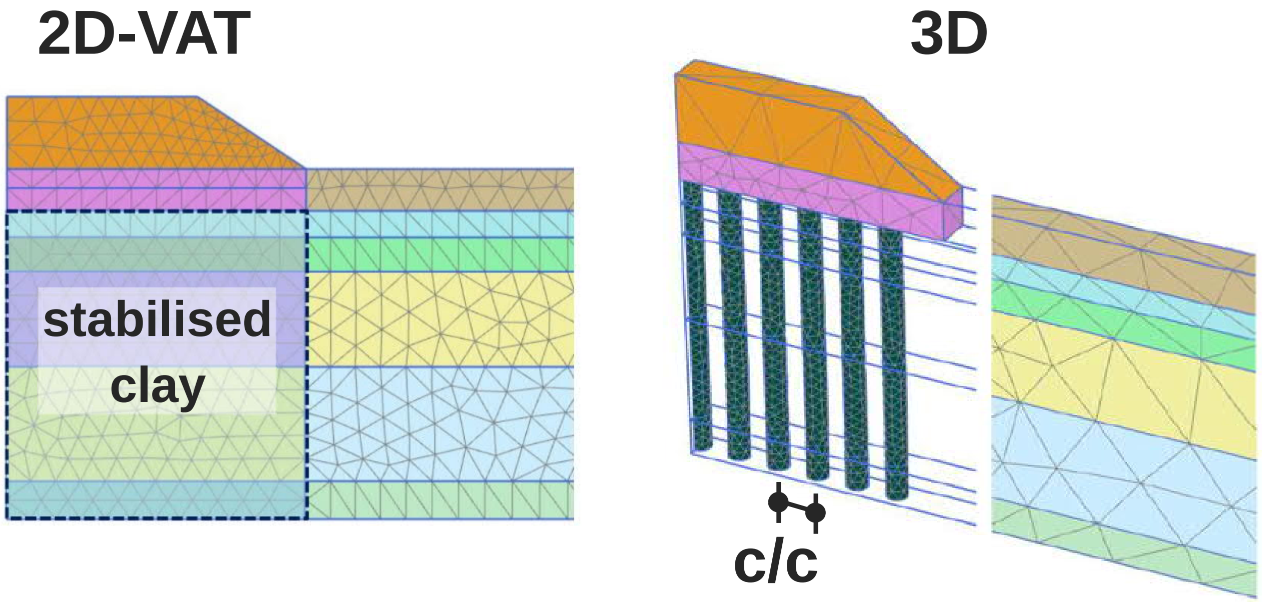
columns and soil (3D) composite system (2D plane strain)

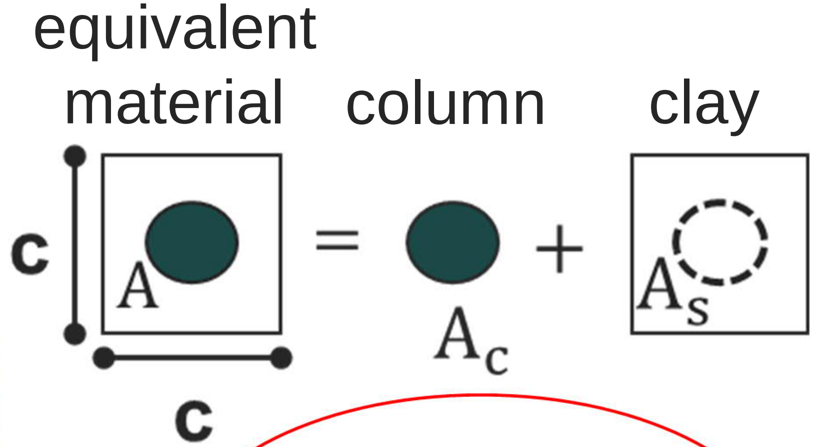
- **Colum/soil system response**
- **Use the known behaviour of constituents (soil and column)**
- **Computationally efficient/fast**

Volume **A**veraging **T**echnique **(VAT)**



VAT for embankments





volume fraction: Ω

$$\Omega_{c,s} = \frac{A_{c,s}}{A}$$

$$\dot{\sigma}^{eq} = \Omega_s \dot{\sigma}^{s} + \Omega_c \dot{\sigma}^{c}$$

$$\dot{\varepsilon}^{eq} = \Omega_s \dot{\varepsilon}^{s} + \Omega_c \dot{\varepsilon}^{c}$$

Schweiger & Pande (1986)
Lee & Pande (1998)

Implementation (Vogler 2009)

PLAXIS2D

finite element code

stiffness matrix **D** $(f(\boldsymbol{\sigma}', \boldsymbol{\kappa}))$ ←

previous stress state $\boldsymbol{\sigma}'^{n-1}$

strain increment $\dot{\boldsymbol{\varepsilon}}$

state variables →

updated stress state $\boldsymbol{\sigma}'^{n}$

updated state variables ←

(for every integration point)

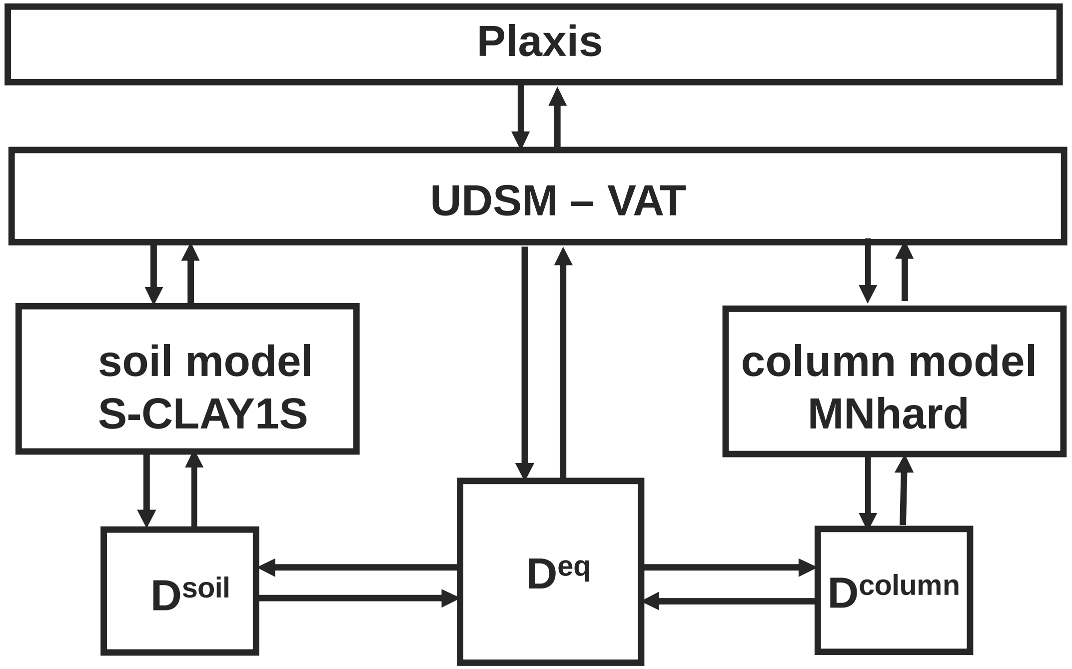
UDSM

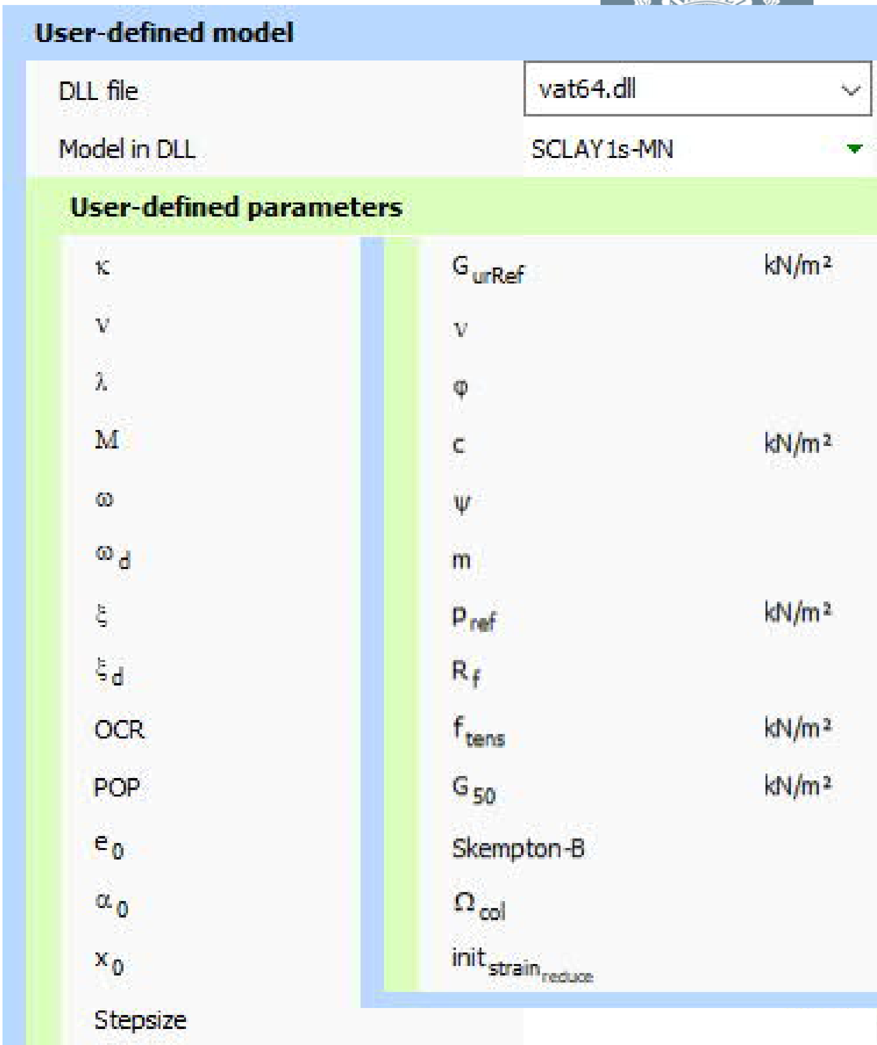
VAT subroutines

- S-CLAY1S
- MNhard
- Averaged material

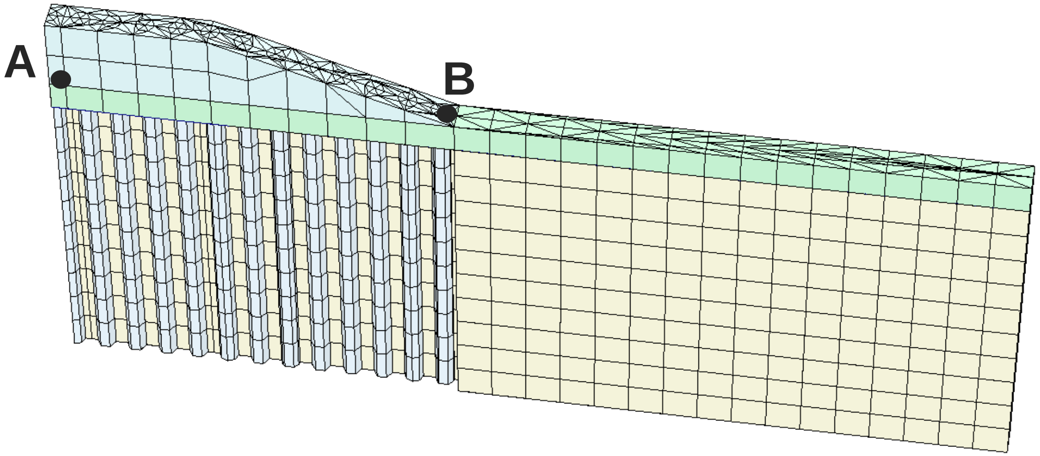
User defined soil model (UDSM)

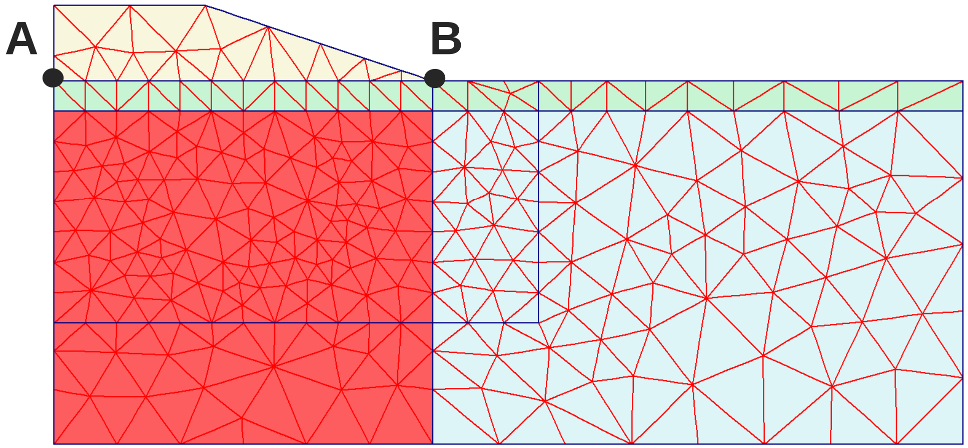
Volume **A**veraging **T**echnique (**VAT**)





FE discretisation - 3D vs 2D

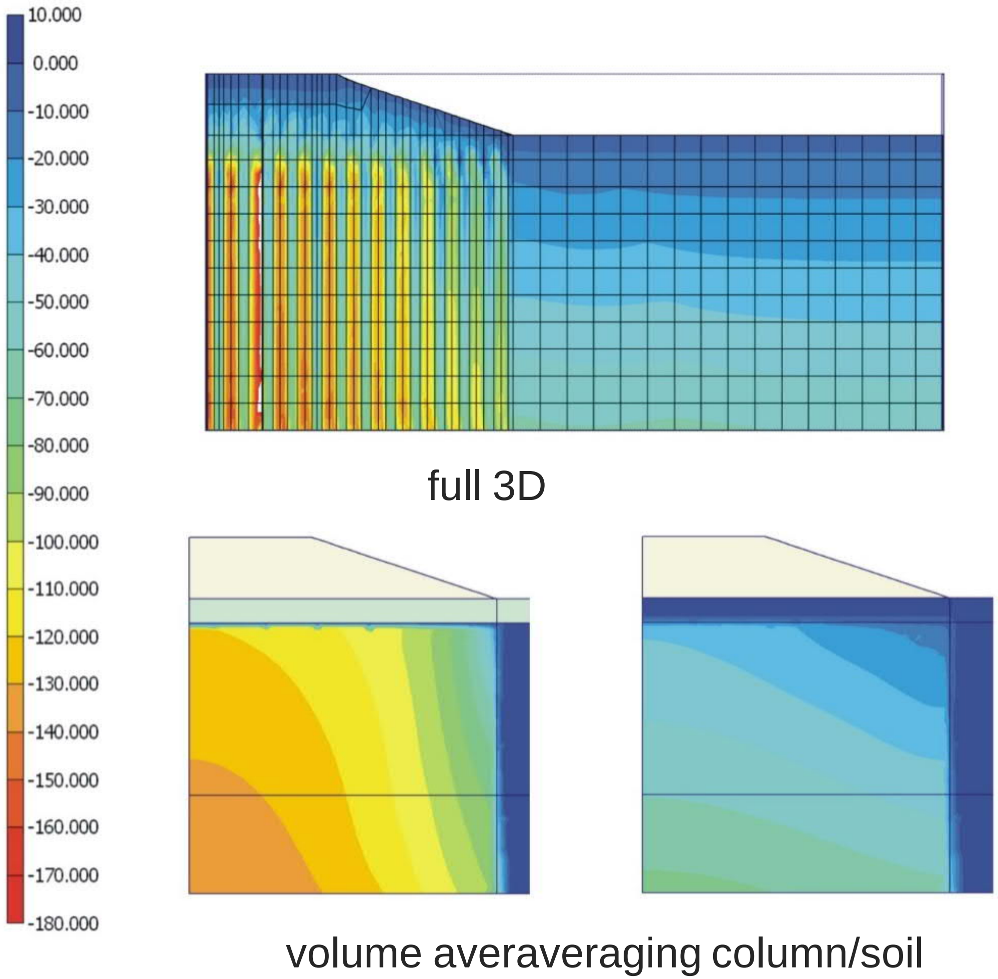




- about 5000 15-noded wedge elements (43000 dof)
- modelling discrete columns in centre to centre wide strip

- about 500 15-noded triangular elements (8000 dof)
- modelling column improved area with volume averaging

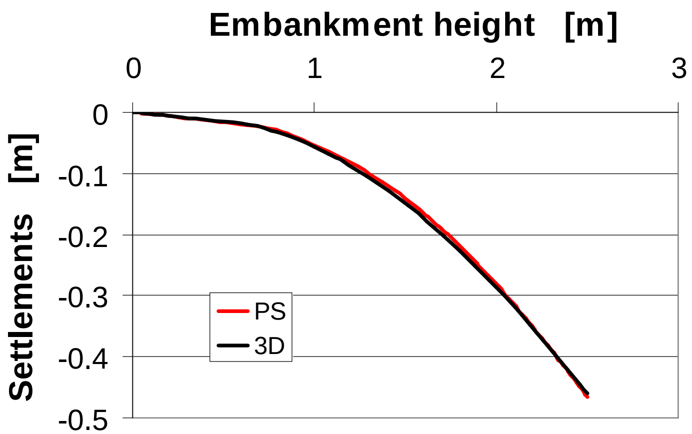
Stress distribution soil/column



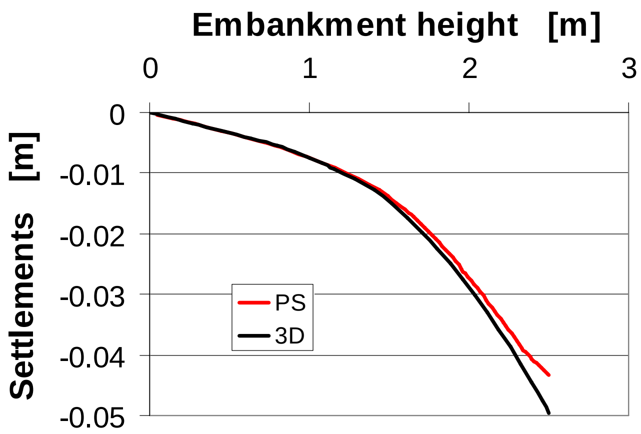
- Vertical effective stress distribution:
- Higher stress in centre of columns for full 3D calculations

Load-displacement curves

centreline **A**



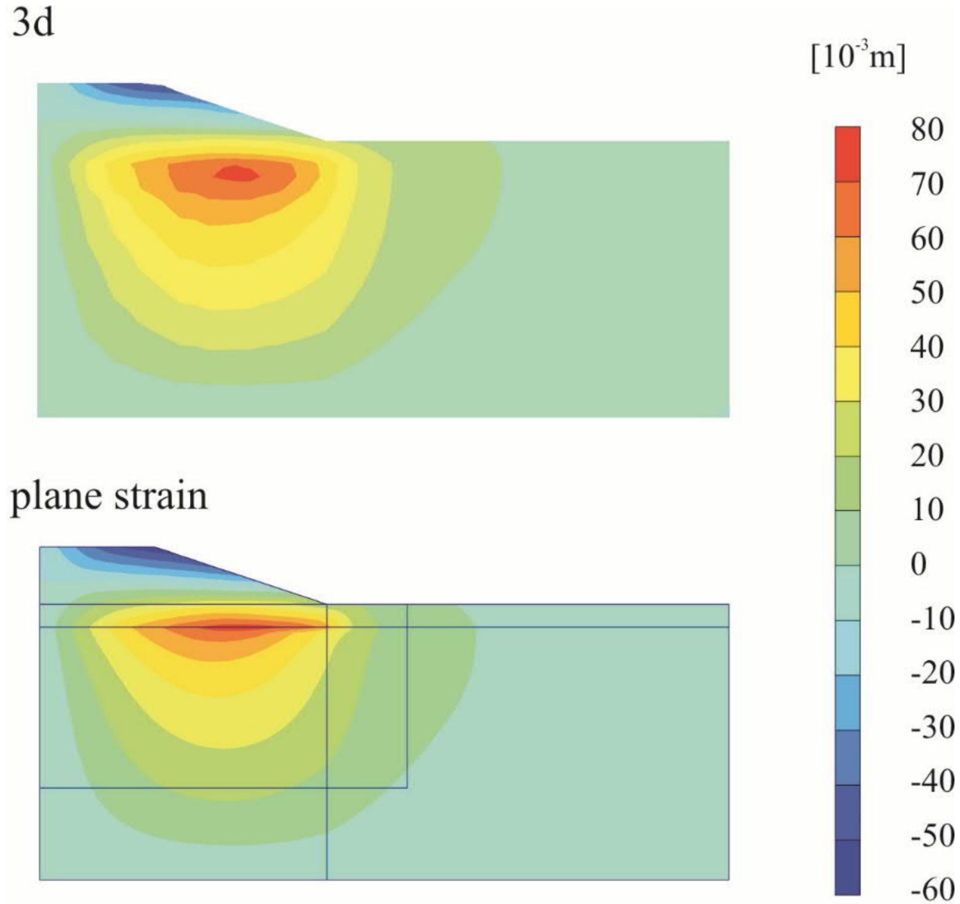
toe **B**



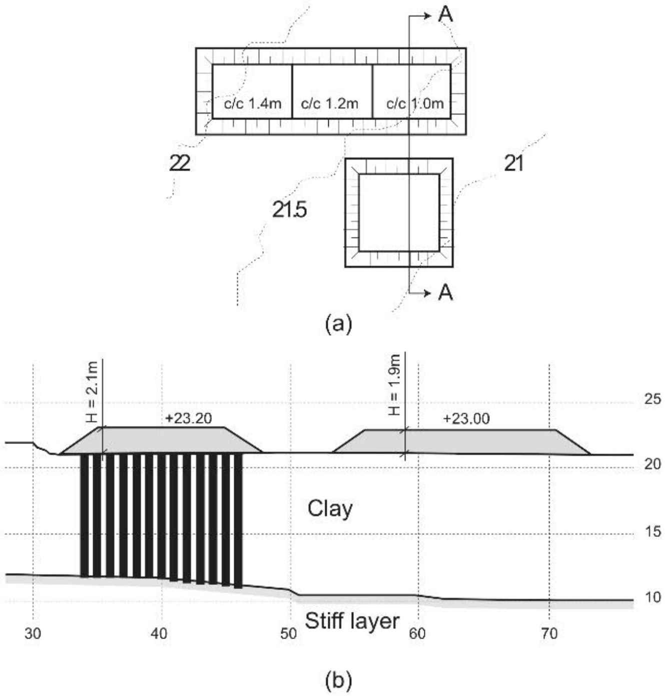
- very good match
- slight deviation under toe

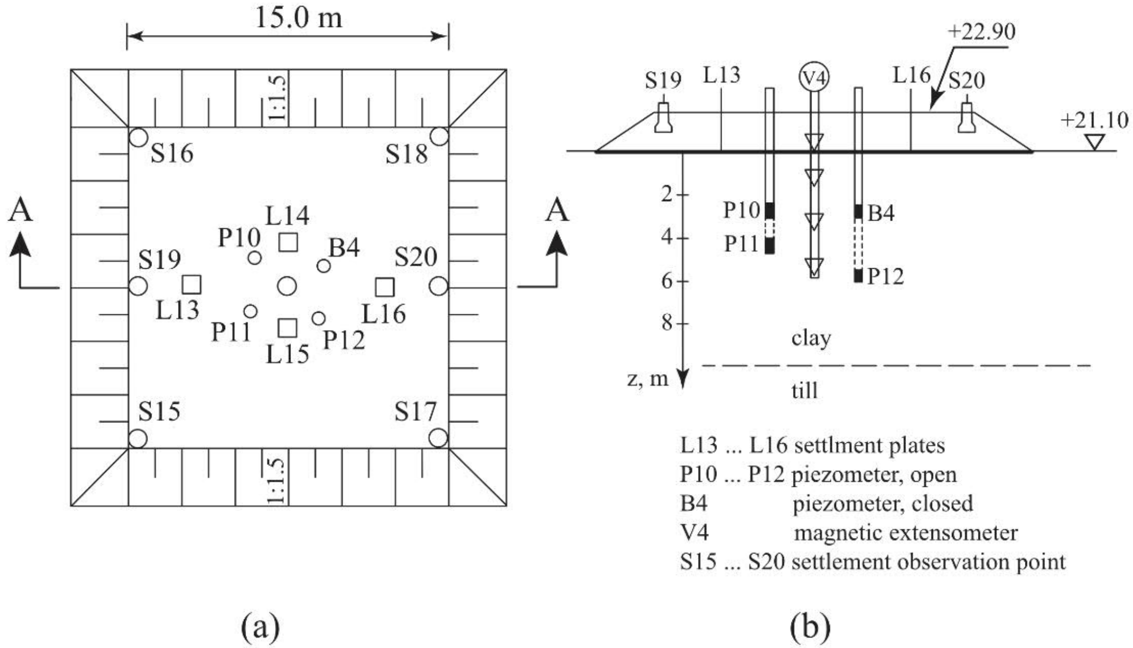
Horizontal displacements

- total horizontal displace-
  ments of 77 mm
- very similar displacement patterns



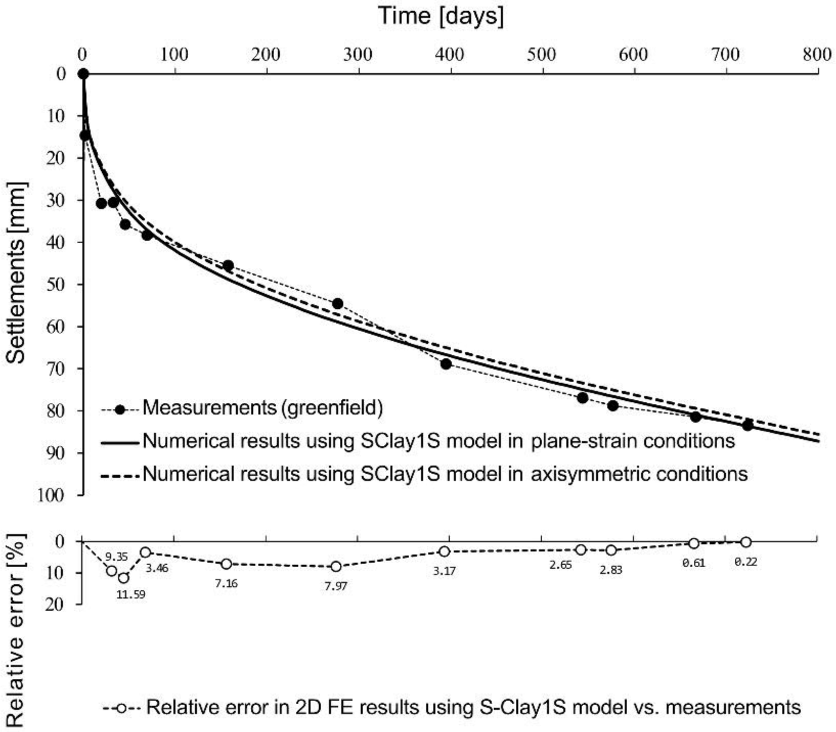
Application: Paimio test embankments

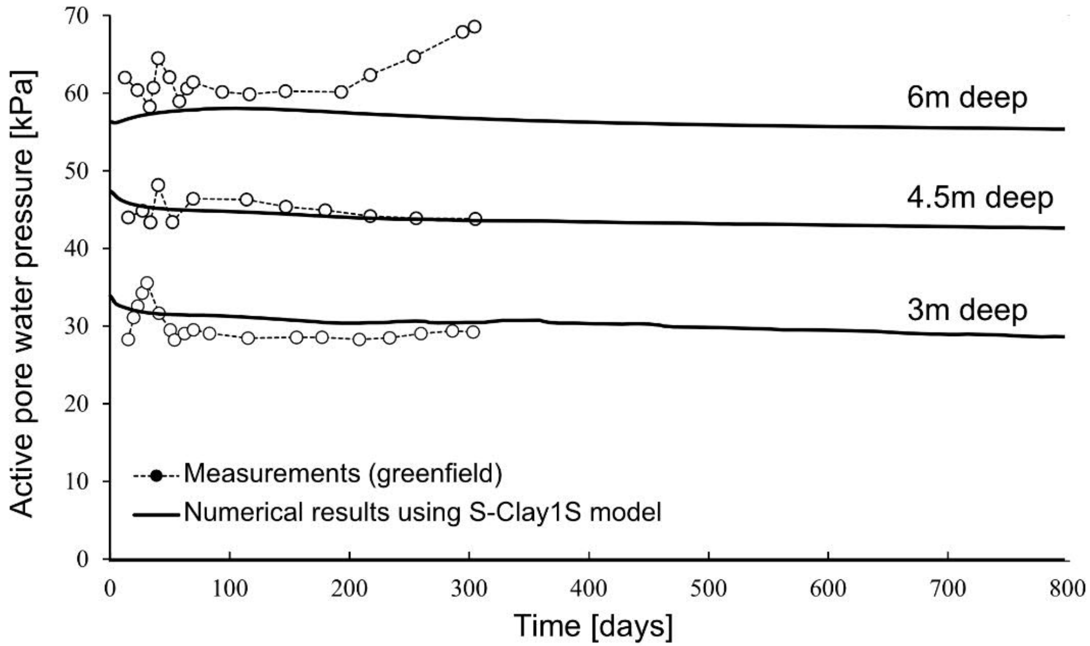




Abed et al. (2025)

Application: Paimio test embankments





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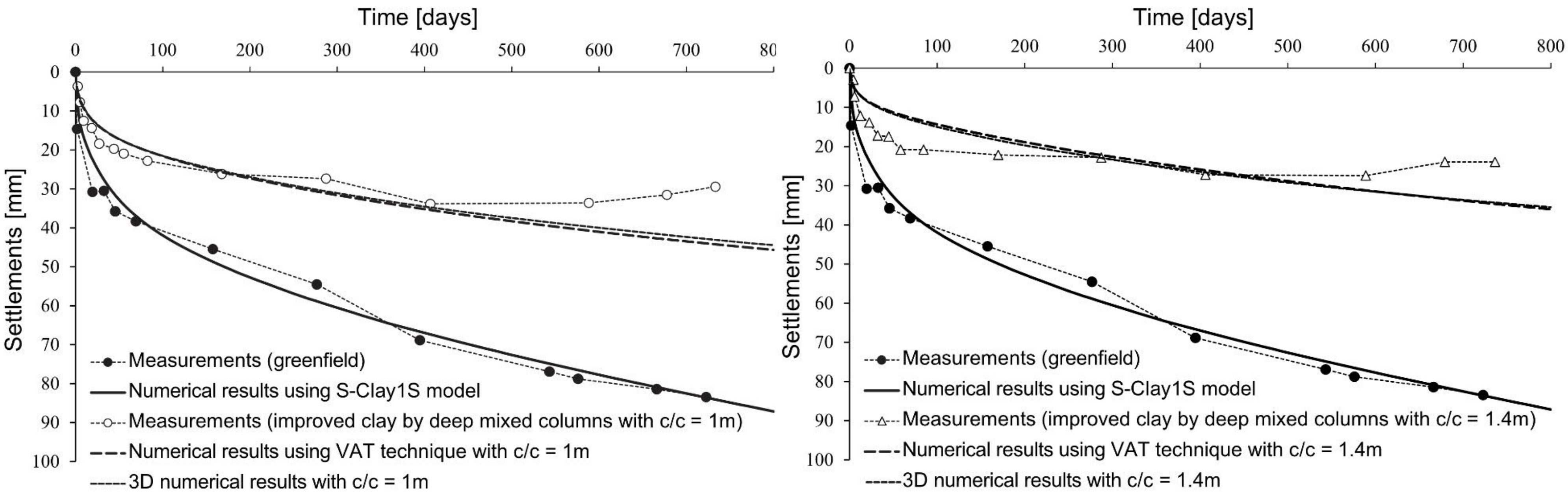
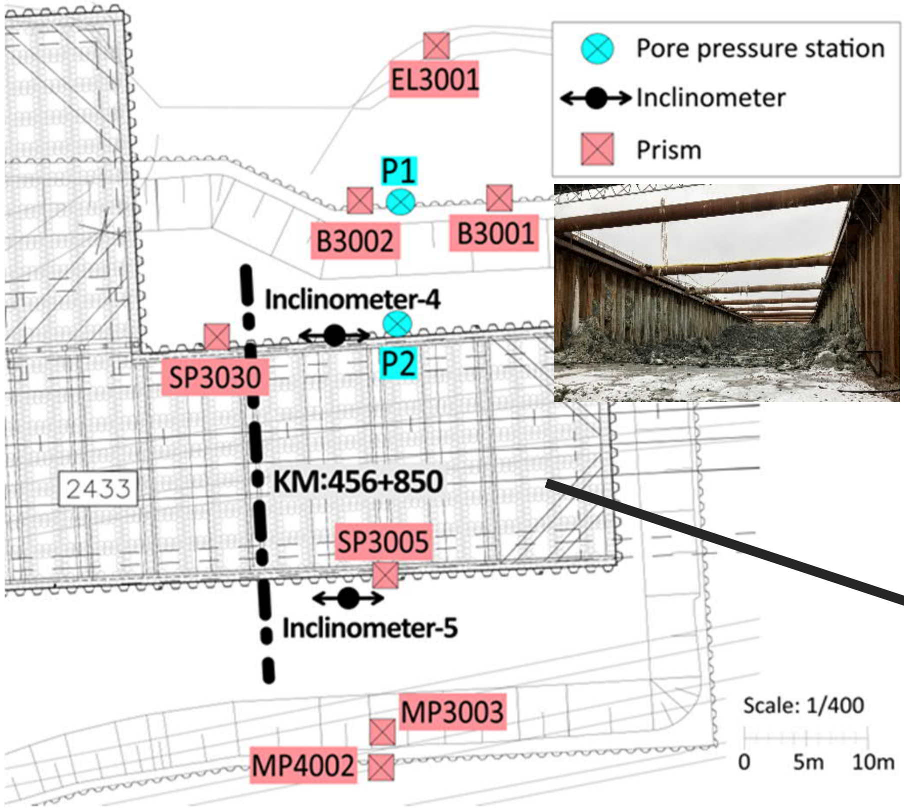
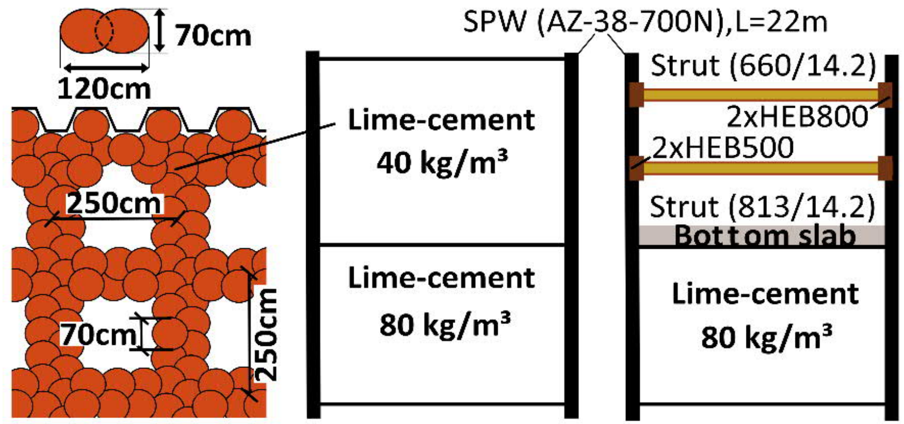
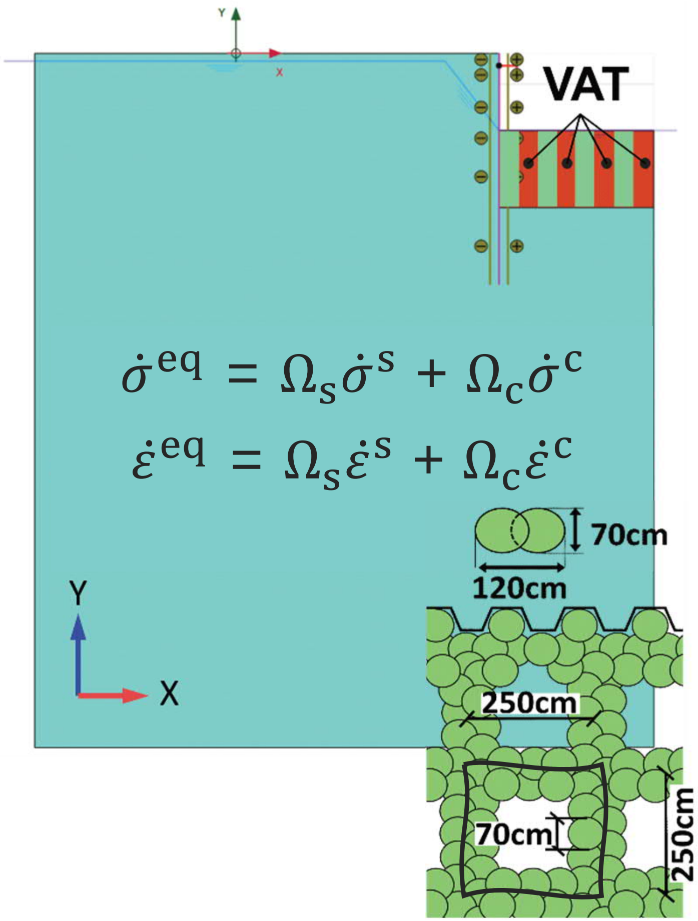


Gothenburg central station





VAT for excavations



$$\dot{\sigma}^{eq} = \Omega_s \dot{\sigma}^{s} + \Omega_c \dot{\sigma}^{c}$$

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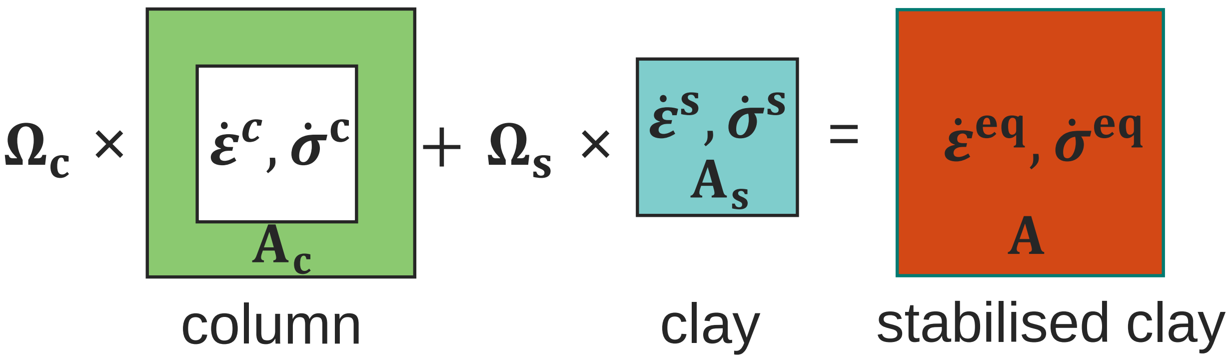


column fraction : $\Omega_c = \frac{A_c}{A}$

clay fraction : $\Omega_s = 1 - \Omega_c$

Equivalent stiffness matrix

$$\mathbf{D}^{\mathbf{eq}} = \Omega_s \mathbf{D}^s \mathbf{S}_1^s + \Omega_c \mathbf{D}^c \mathbf{S}_1^c$$

$$\mathbf{S}_1^{s,c} = f\,(\Omega_s/\Omega_c, \mathbf{D}^s, \mathbf{D}^c)$$

Kinematic constraints:

$$\Delta\varepsilon_{xx}^{eq} = \Delta\varepsilon_{xx}^{c} = \Delta\varepsilon_{xx}^{s}$$

$$\Delta\varepsilon_{zz}^{eq} = \Delta\varepsilon_{yy}^{c} = \Delta\varepsilon_{yy}^{s}$$

$$\Delta\gamma_{zx}^{eq} = \Delta\gamma_{zx}^{c} = \Delta\gamma_{zx}^{s}$$

Local equilibrium:

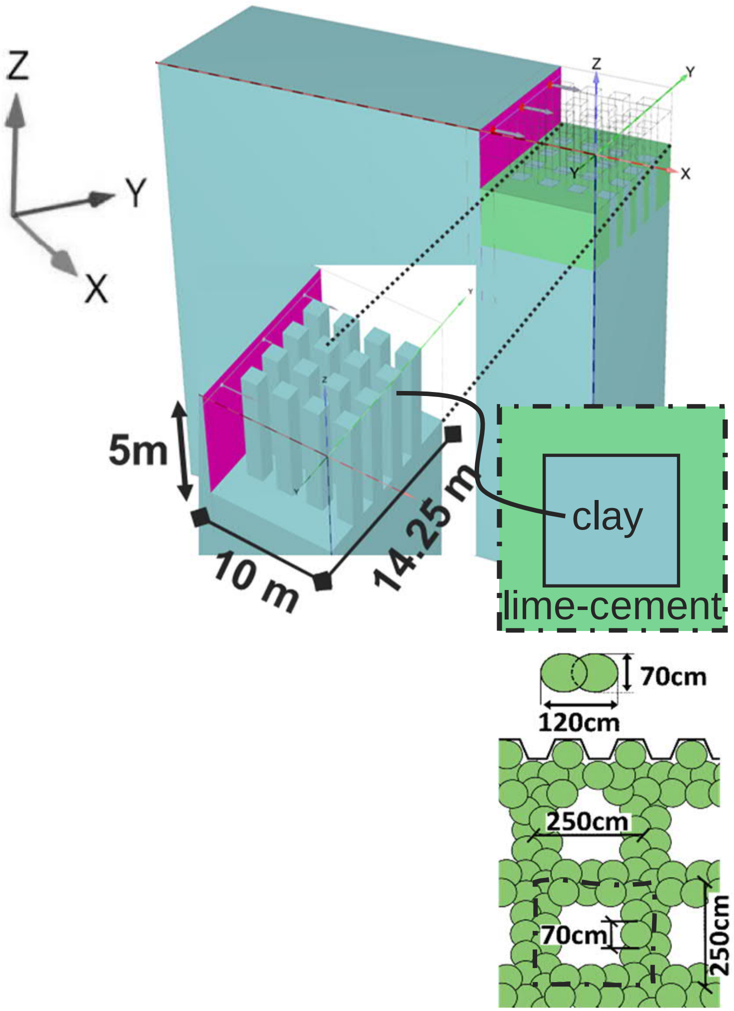
$$\Delta\sigma_{yy}^{eq} = \Delta\sigma_{yy}^{c} = \Delta\sigma_{yy}^{s}$$

$$\Delta\tau_{xy}^{eq} = \Delta\tau_{xy}^{c} = \Delta\tau_{xy}^{s}$$

$$\Delta\tau_{yz}^{eq} = \Delta\tau_{yz}^{c} = \Delta\tau_{yz}^{s}$$

⬇

$$\mathbf{D}^{\mathbf{eq}}$$



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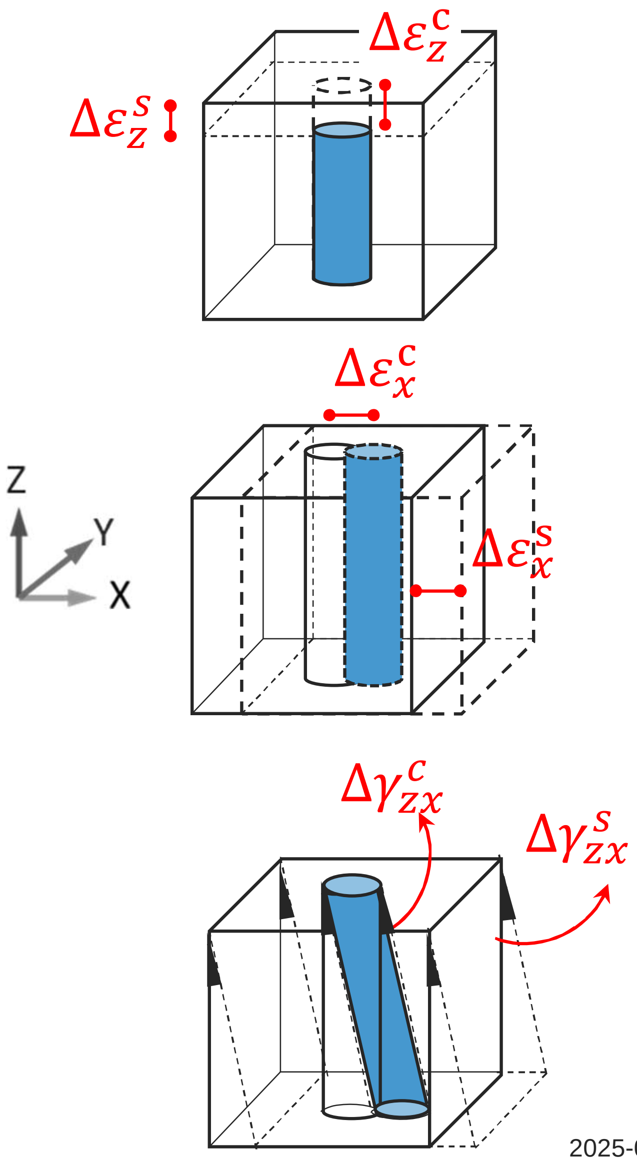

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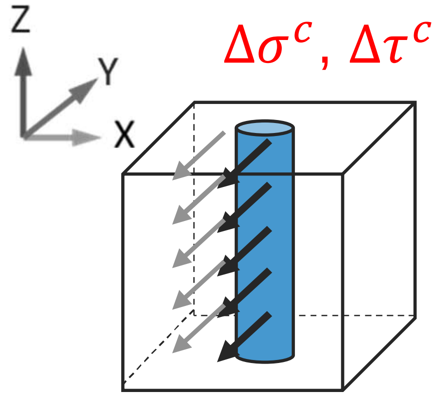

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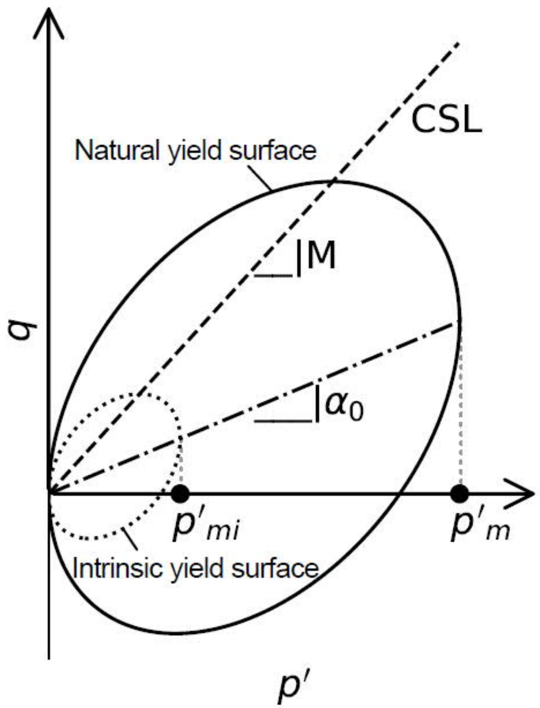
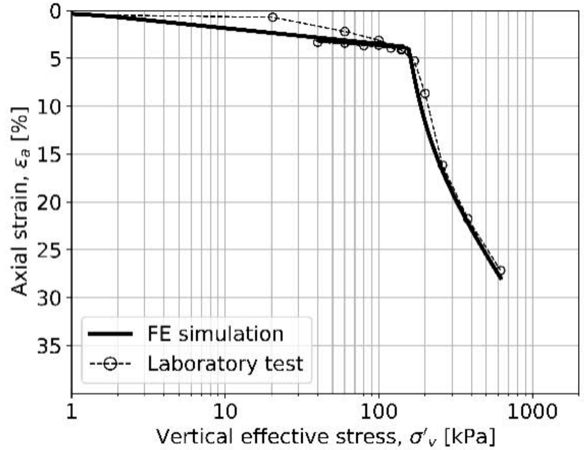
$$\Delta\tau_{yz}^{\mathrm{eq}} = \Delta\tau_{yz}^{\mathrm{c}} = \Delta\tau_{yz}^{\mathrm{s}}$$

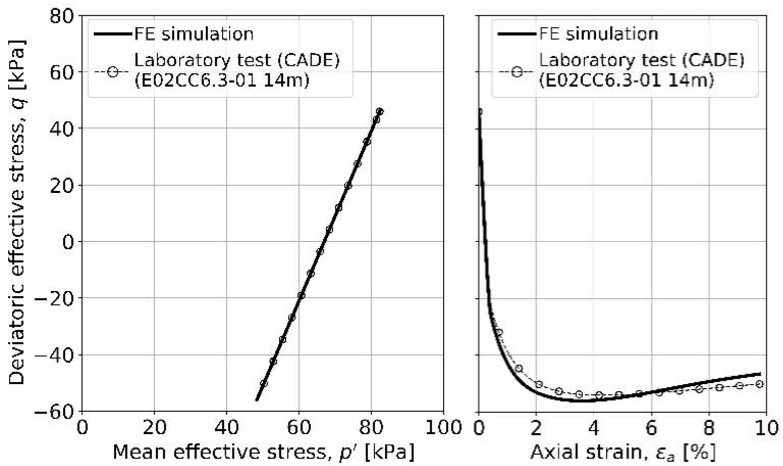
stress corrections

Modelling soft clay: S-CLAY1S



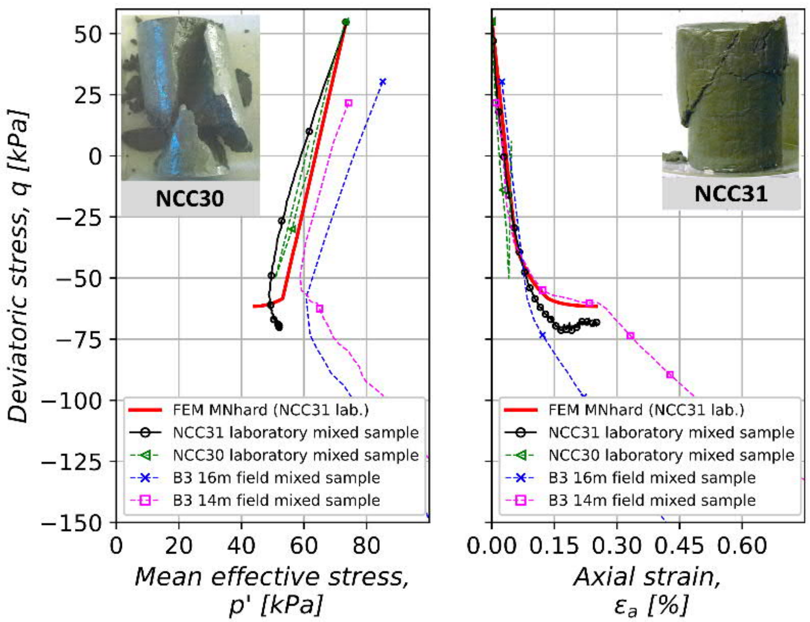
- Volumetric hardening
- Evolution of anisotropy
- Destructuration





Koskinen et al. (2002)
Karstunen et al. (2005)

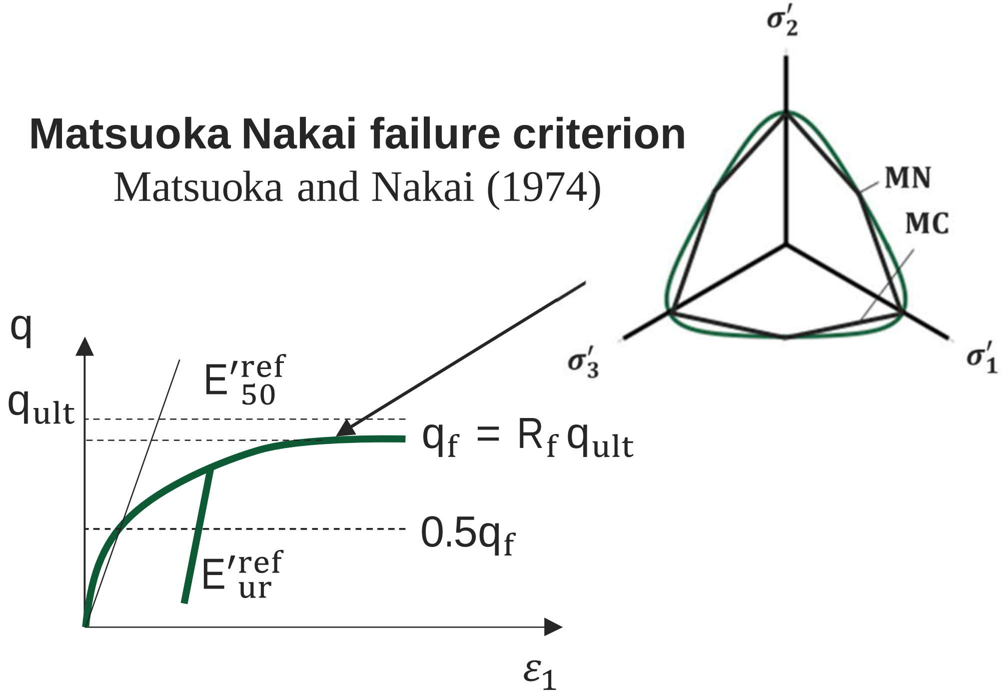
Modelling lime-cement columns: Matsuoka-Nakai hardening (MNhard) model



Bozkurt et al. (2023)

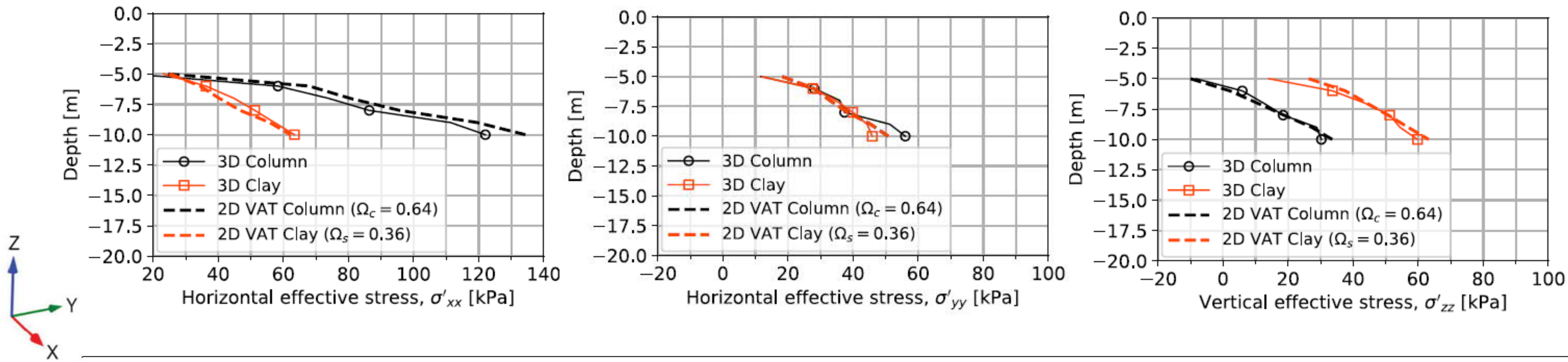
Matsuoka Nakai failure criterion

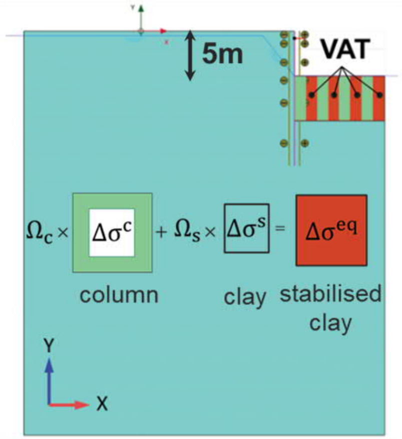
Matsuoka and Nakai (1974)

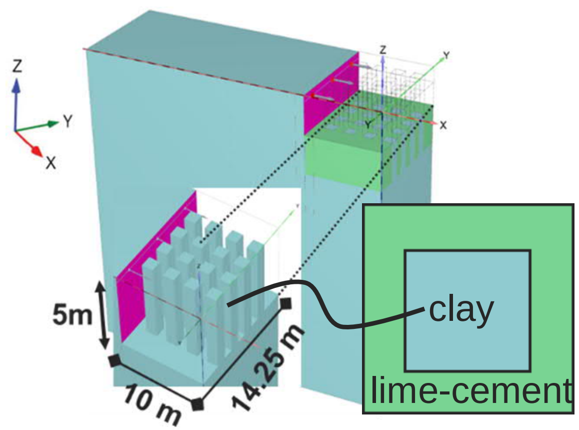


Benz (2007)

Verification of VAT for excavations





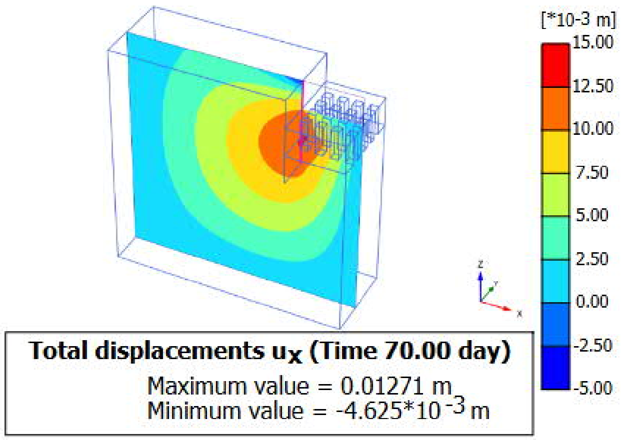


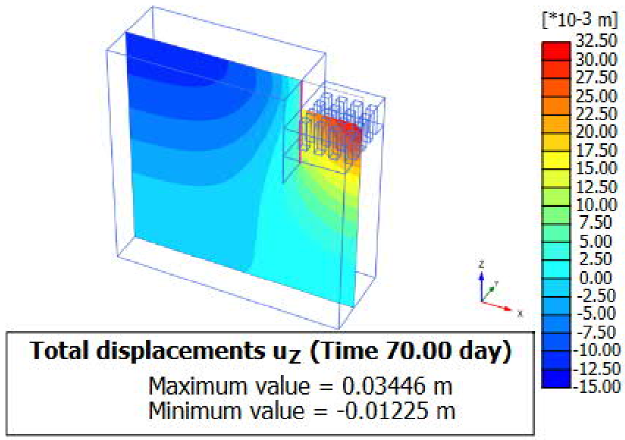
fully coupled consolidation analyses

70 days

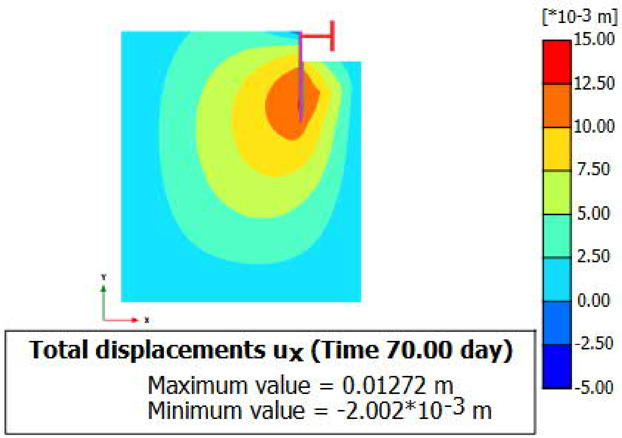
Comparison of 2D and 3D

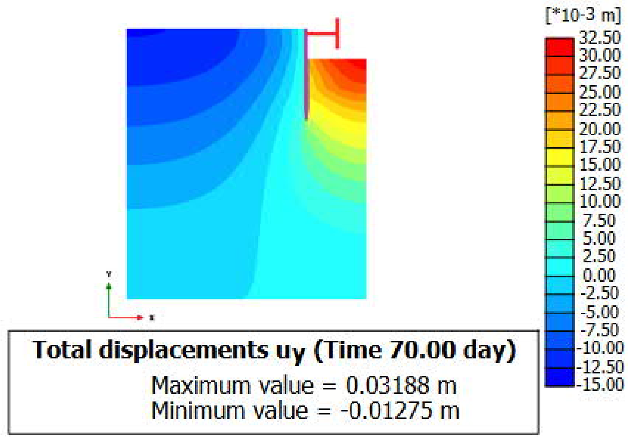
3D





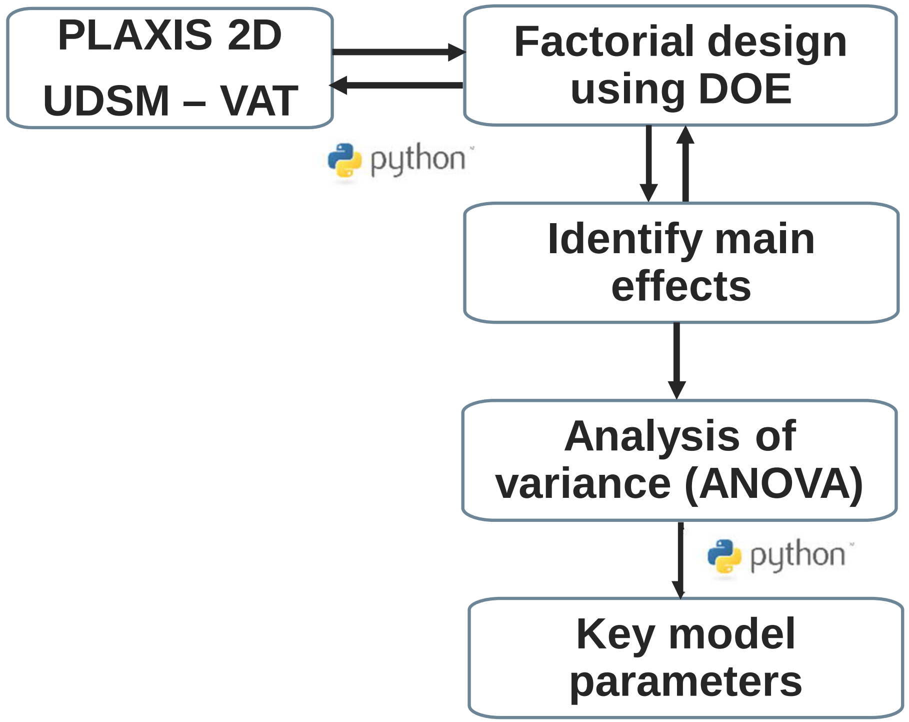
2D-VAT



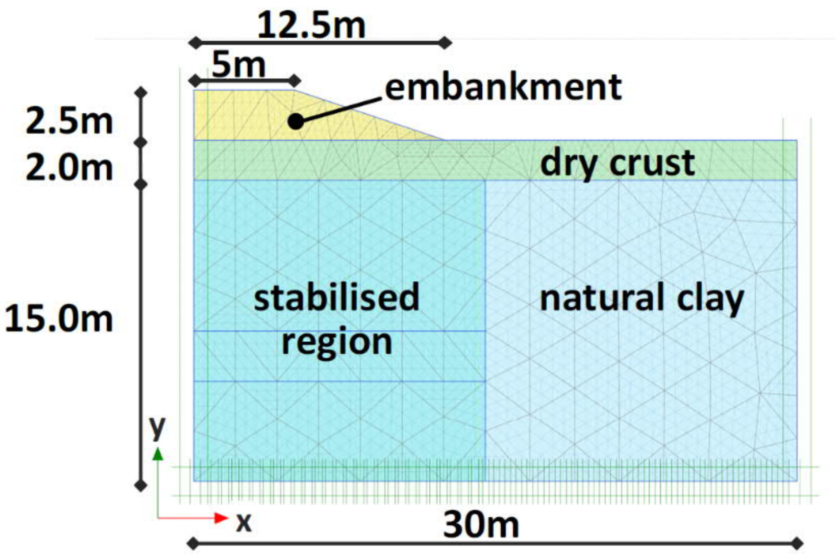


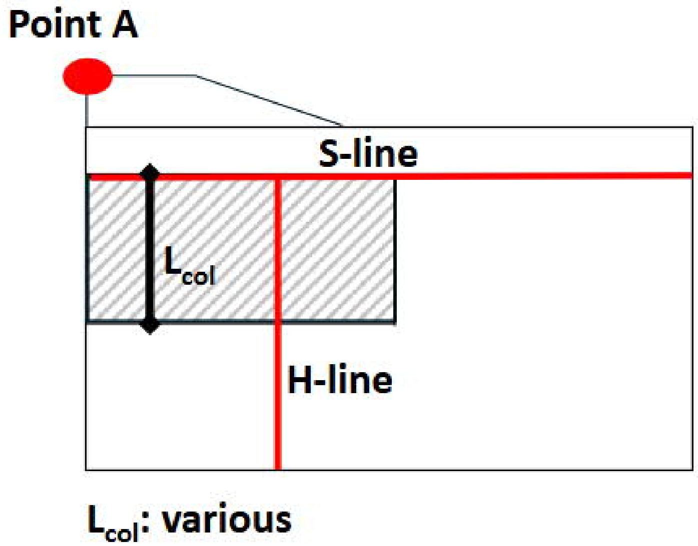
Design of experiments (DOE)

- System-level sensitivity analysis
- Key factors in models with many factors
- The full range of all model parameters
- Interaction effects



Sensitivity analyses (DOE)





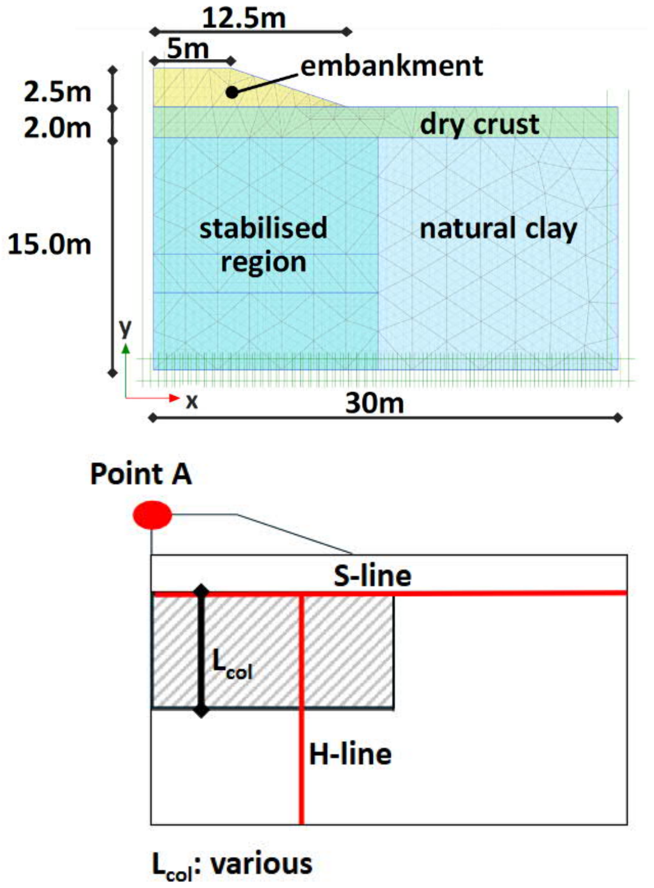
- 2D-VAT and DOE, fully coupled analyses
- fractional factorial design: a range of 5%

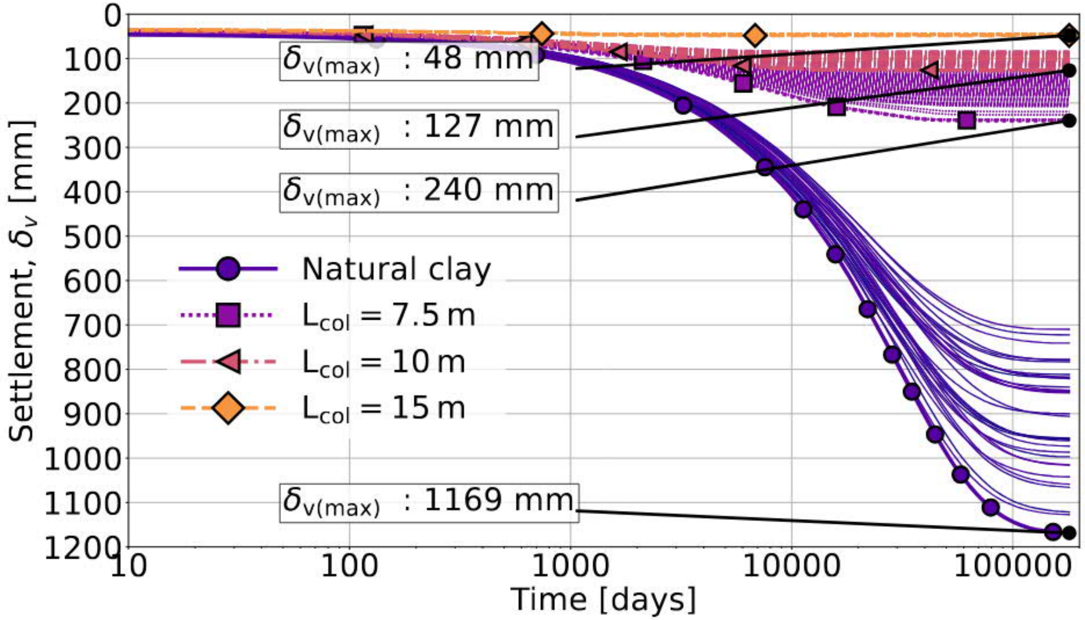
Bozkurt et al. (2025)

- soft clay: S-CLAY1S model
- columns: MNhard model
- various column lengths and spacings

Sensitivity analyses (DOE)

range of factors: 5%



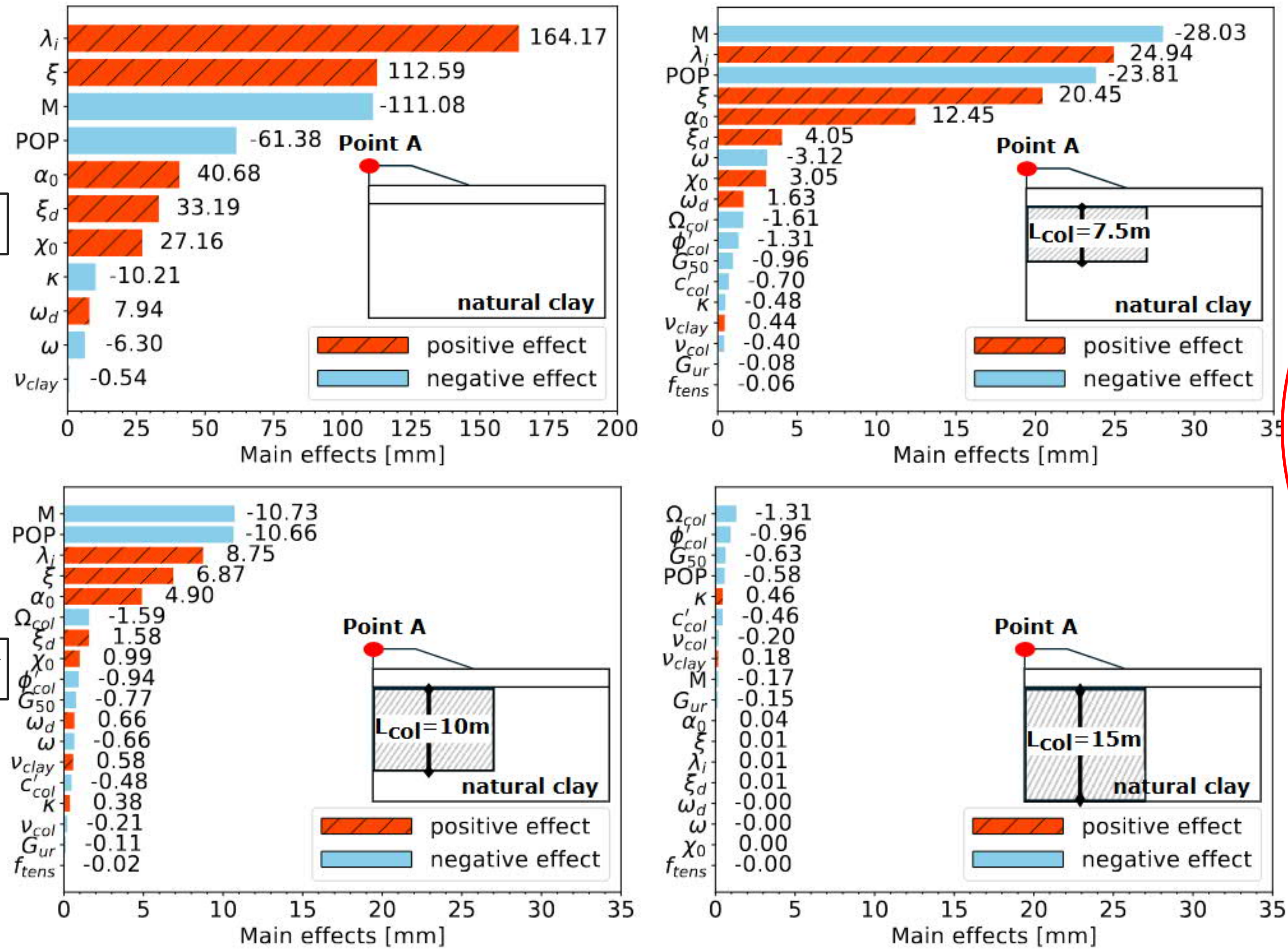


S-line

Influential factors

2^{11-6}_{IV}

2^{18-11}_{IV}



2^{18-1}_{IV}

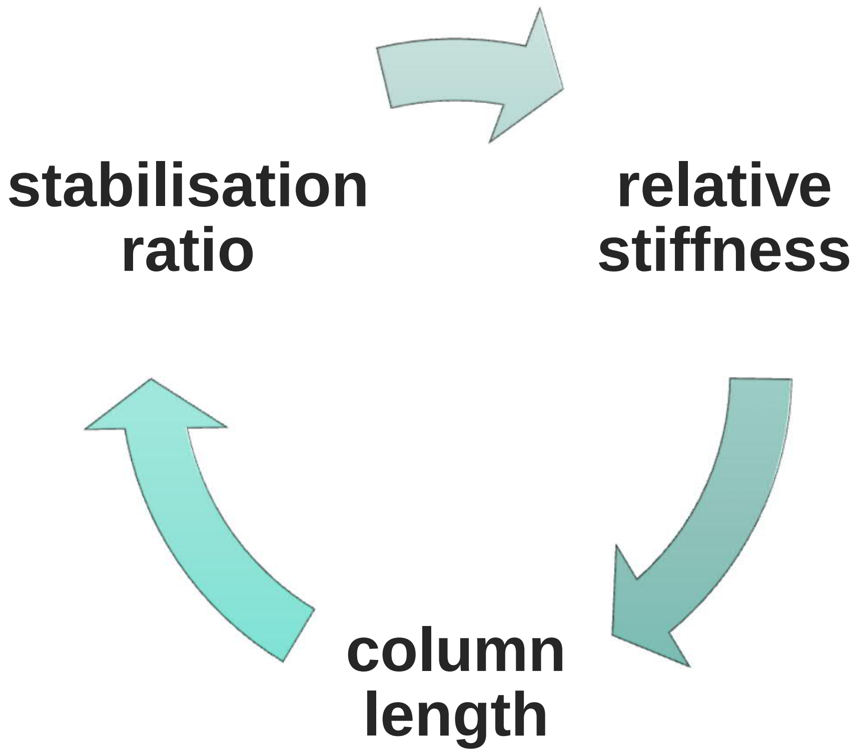
2^{18-1}_{IV}

- End-bearing columns: **Columns govern!** Ω_{col}, G_{50}
- Floating colums: **Soft clay governs!** ***M, λ_i, POP, ξ, α_0***

range of factors: 5%

How weak can the columns be?

column fraction Ω_{col}



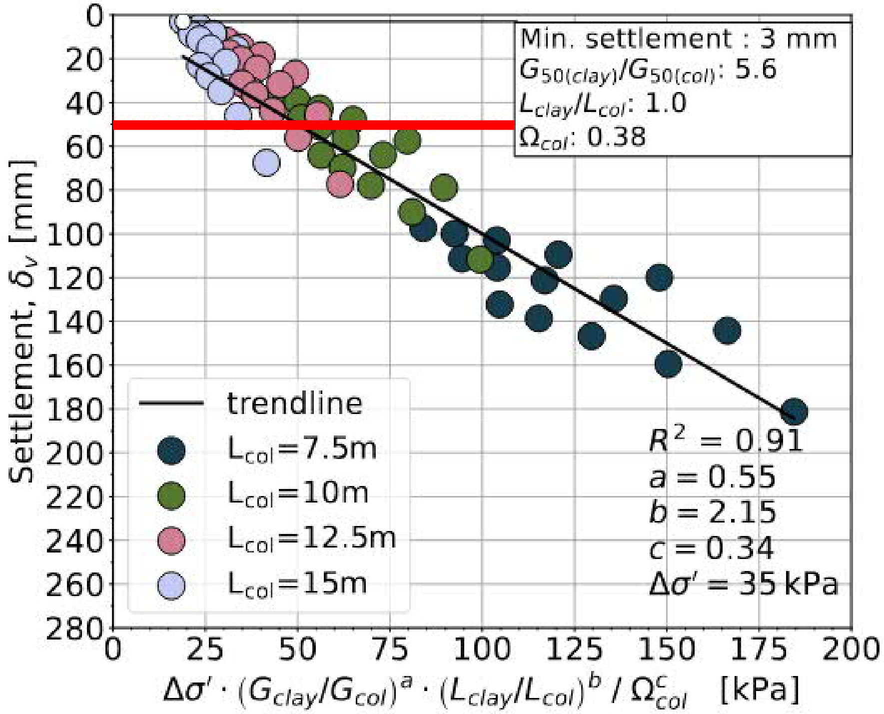
$G_{col} \sim \text{binder}$

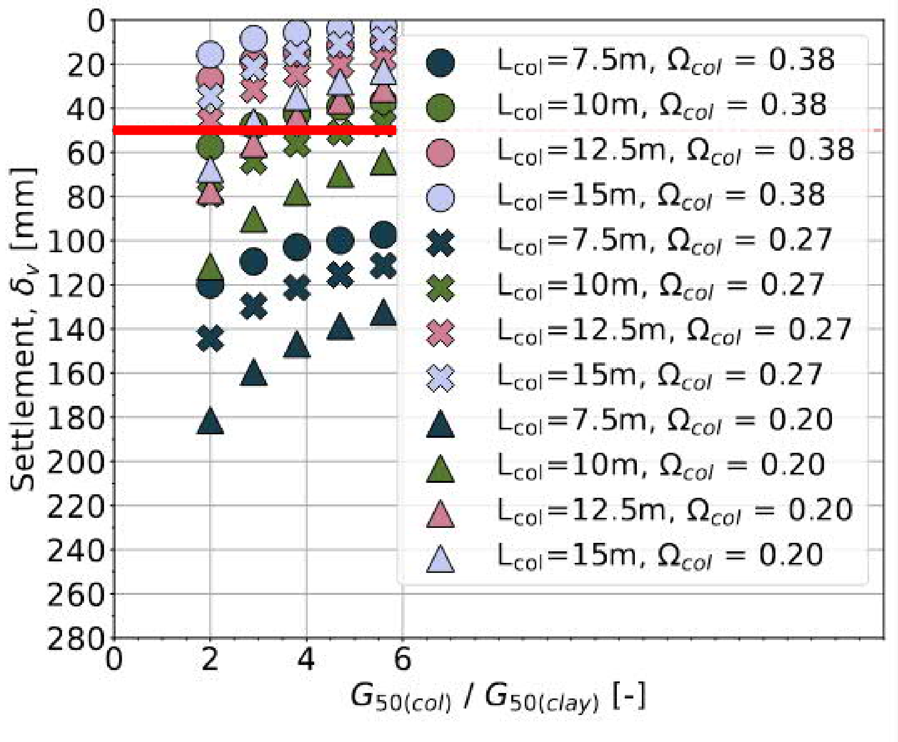
$G_{ratio} = G_{50(col)} / \ G_{50(clay)}$

L_{col}: various

Linking stiffness, length, and stabilisation ratio

Limit settlement: 50mm



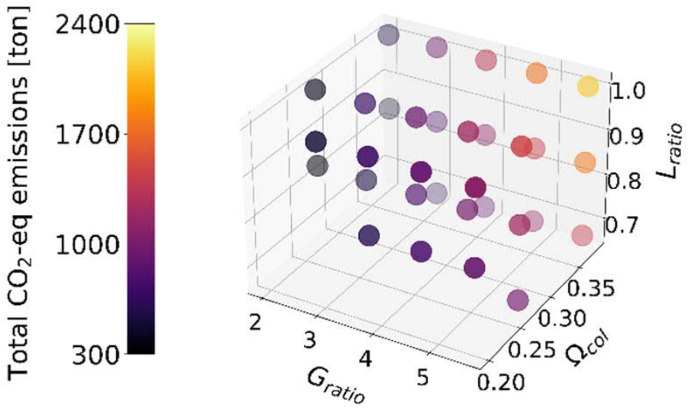


Material use related GHE

Combination	Component	Proportion (%)	Quantity (ton)	CO_2-eq emissions factor (kg CO_2-eq/ton)	Total CO_2-eq emissions (ton CO_2-eq)
Upper boundary G_{ratio}=5.6 (110 kg/m^3)[b] L_{col} = 15 m Ω_{col} = 0.38	QL	50	921	1450[a]	1335
	OPC	50	921	870[a]	801
	Total	**100**	**1842**	**1842**	**2136**
Lower boundary G_{ratio}=2.0 (30 kg/m^3)[b] L_{col} = 12.5 m Ω_{col} = 0.27	QL	50	144	1450[a]	208
	OPC	50	144	870[a]	125
	Total	**100**	**288**	**2320**	**334**

[a] CO_2-eq emissions factors are based on Klimatkalkyl version 7.0 by Trafikverket (Swedish Transport Administration).

[b] Binder contents corresponding to target stiffness moduli are estimated based on Paniagua et al. (2021).



$$\mathbf{G_{ratio} = G_{50(col)}/G_{50(clay)}}$$

column fraction: $\mathbf{\Omega_{col}}$

Conclusions

- For most geotechnical problems on deep mixed columns, serviceability limit controls the design.
- Volume Averaging Technique (VAT) enables mapping of a periodically arranged column system to 2D, yet modelling the individual constituent (clay and columns) separate.
- VAT formulations were developed for two cases: embankments and excavations, and verified against 3D simulations.
- VAT can significantly save computational time, yet similar results to those of 3D analysis.
- The systematic approach, using the VAT in combination with factorial design, can be used to perform efficiently sensitivity analyses and/or data-driven procedures in a plane strain analysis.

Future studies

- Incorporating VAT with a probabilistic method, such as the Random Finite Element method, helps account for the spatial variability of the properties of both natural clay and stabilised clay.
- VAT can be implemented to investigate the creep rate-dependent response of stabilised clay using appropriate constitutive soil models.

Key references for VAT

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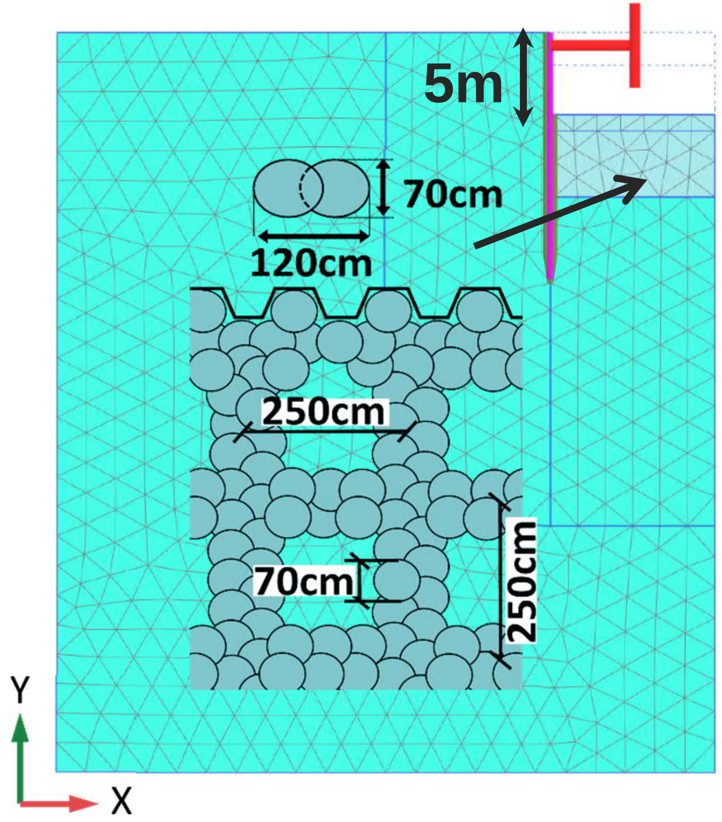
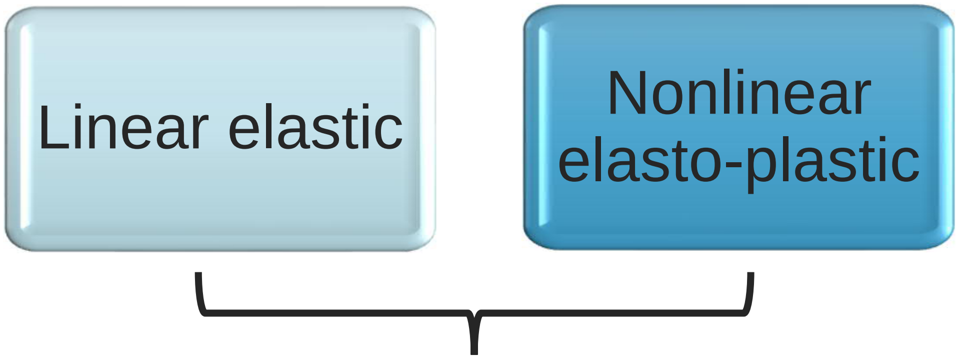
Stiffness reduction methods

$$E'_{\mathrm{composite}} = a_{\mathrm{ratio(column)}} \times E'_{\mathrm{column}} + (1 - a_{\mathrm{ratio(column)}}) \times E'_{\mathrm{clay.avg}}$$

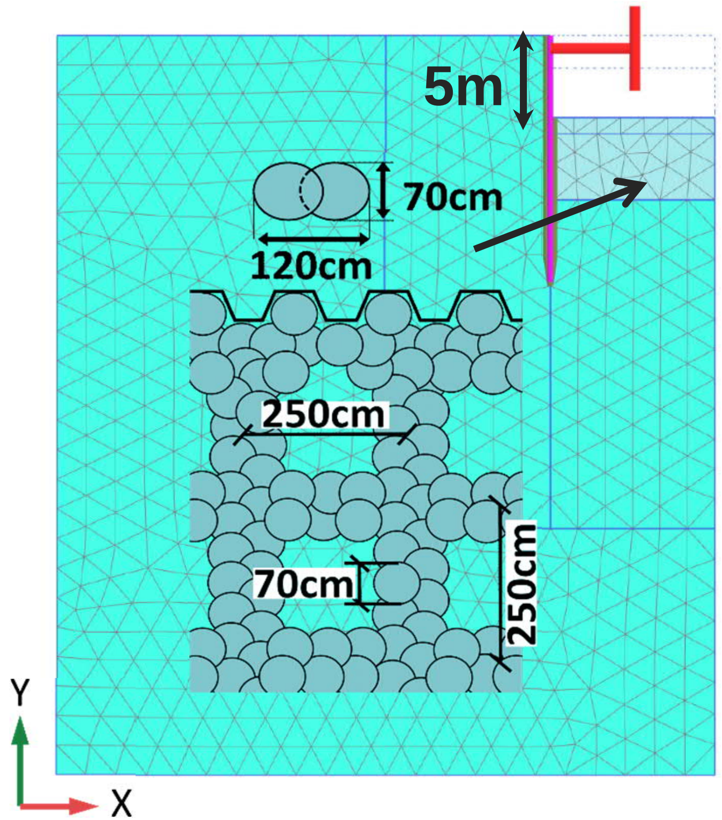


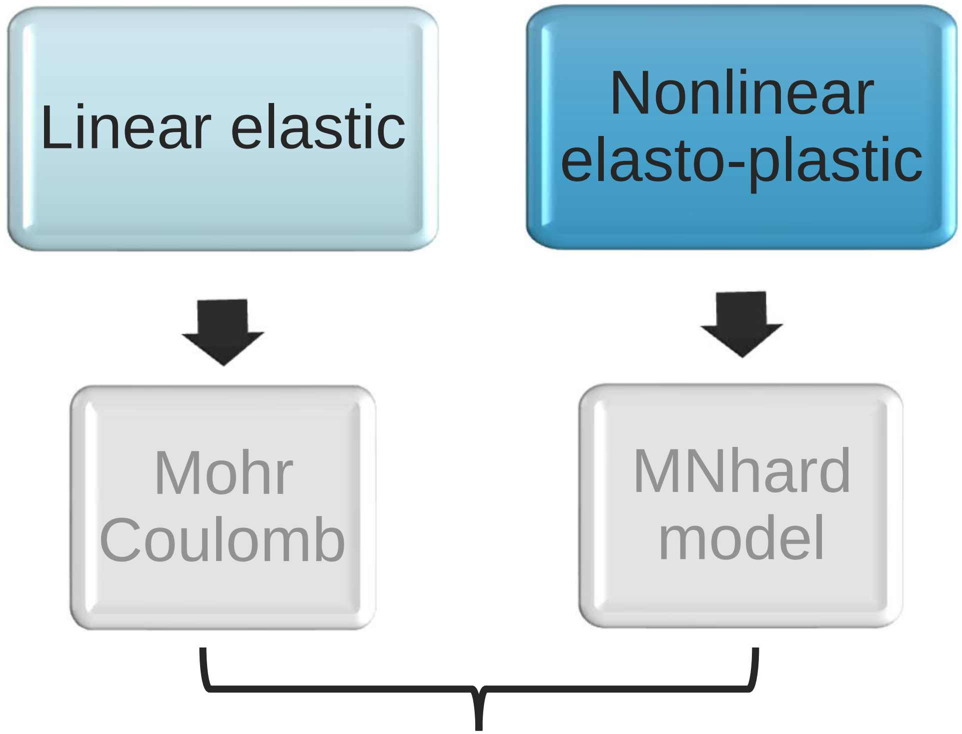


$$E'_{\mathrm{composite}}, c'_{composite}, \phi'_{composite}$$

Stiffness reduction methods

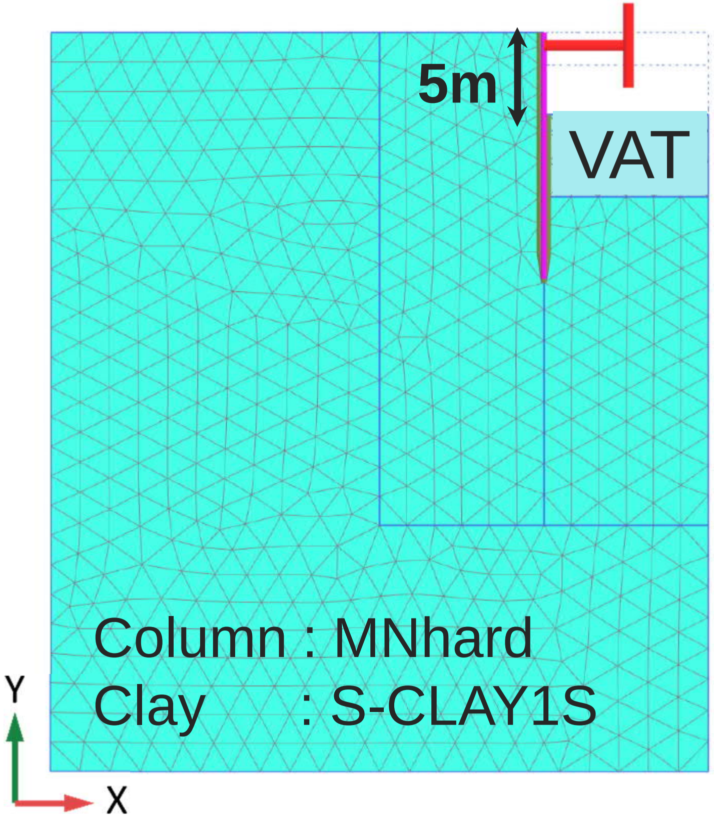
$$E'_{\text{composite}} = a_{\text{ratio(column)}} \times E'_{\text{column}} + (1 - a_{\text{ratio(column)}}) \times E'_{\text{clay.avg}}$$

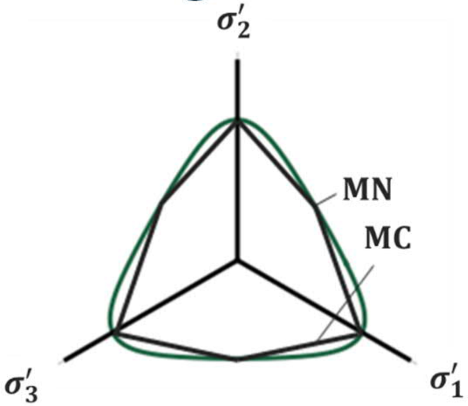




$$E'_{\text{composite}}, c'_{composite}, \phi'_{composite}$$

Advanced numerical analysis





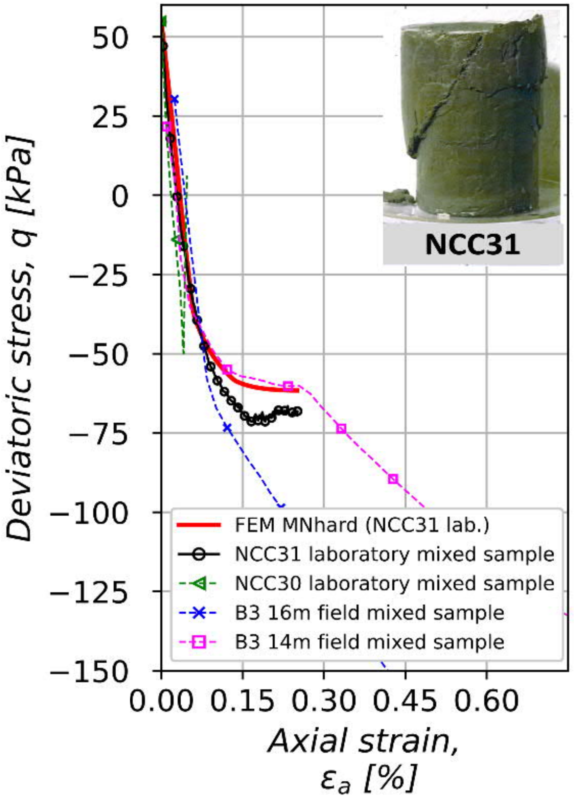
Mohr Coulomb

MNhard model

$E'_{composite}, c'_{composite}, \phi'_{composite}$

Volume averaging technique (VAT)

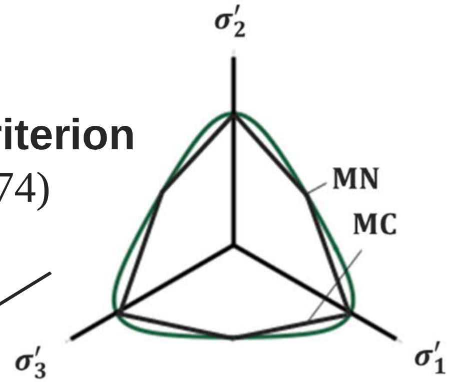
Modelling lime-cement columns: Matsuoka-Nakai Hardening (MNhard) model

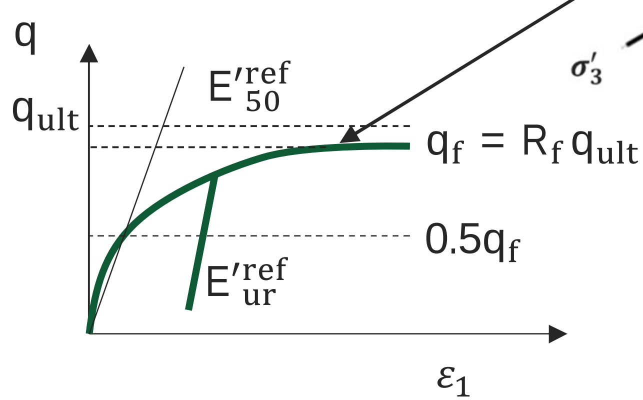


Bozkurt et al. (2023)

Matsuoka Nakai failure criterion

Matsuoka and Nakai (1974)

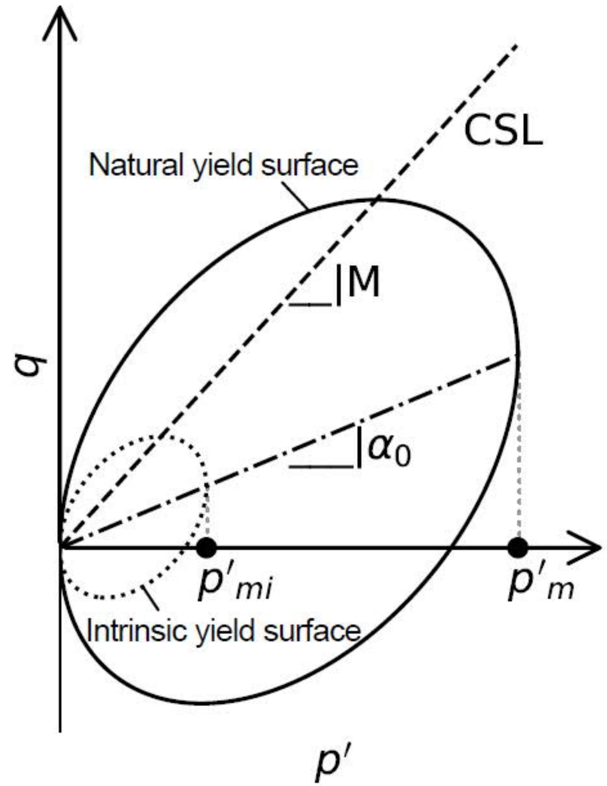




$$E'_{50} = E'^{\ ref}_{50} \left(\frac{\sigma'_3 + c' \cot \phi'}{\sigma'^{\ ref} + c' \cot \phi'} \right)^m$$

$$E'_{ur} = E'^{\ ref}_{ur} \left(\frac{\sigma'_3 + c' \cot \phi'}{\sigma'^{\ ref} + c' \cot \phi'} \right)^m$$

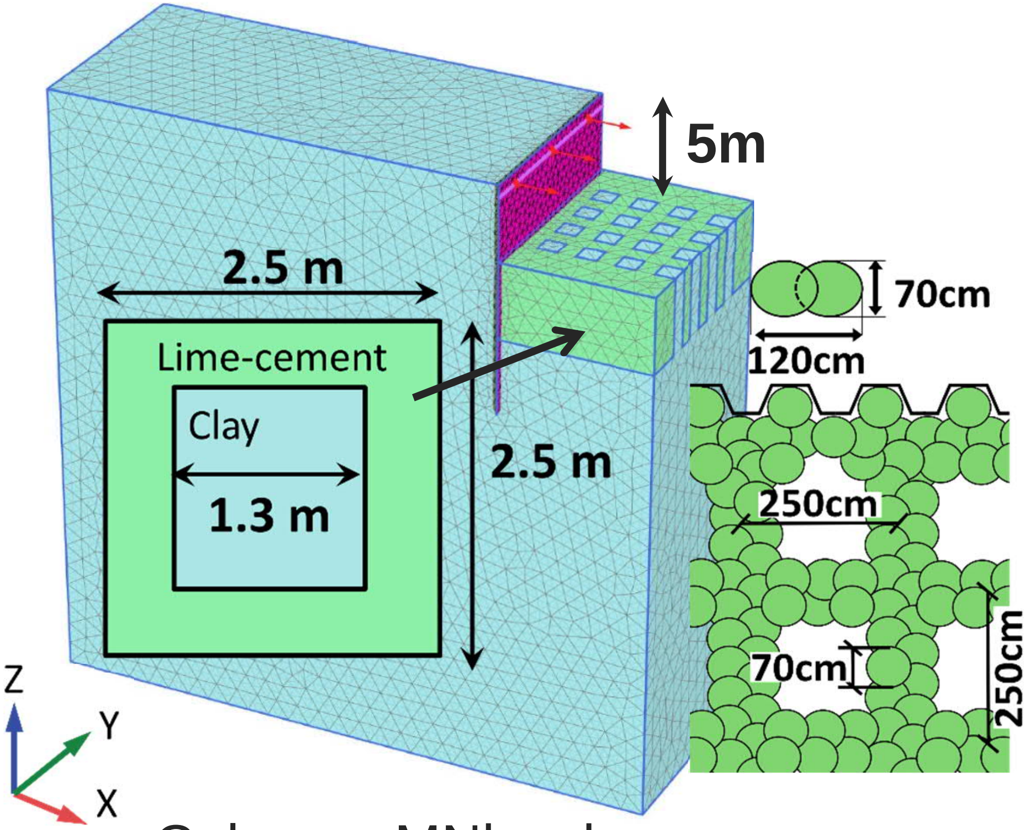
Modelling soft clay: S-CLAY1S



- Volumetric hardening
- Evolution of anisotropy
- Destructuration

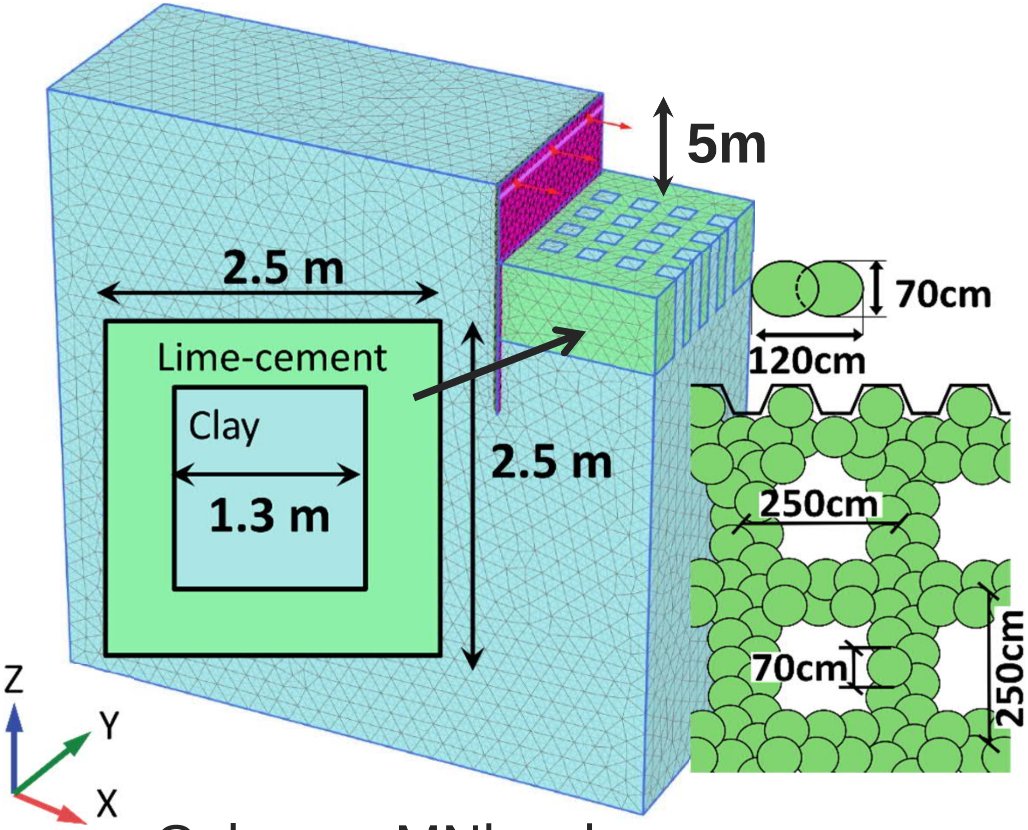
Karstunen et al. (2005)

Volume averaging technique (VAT)

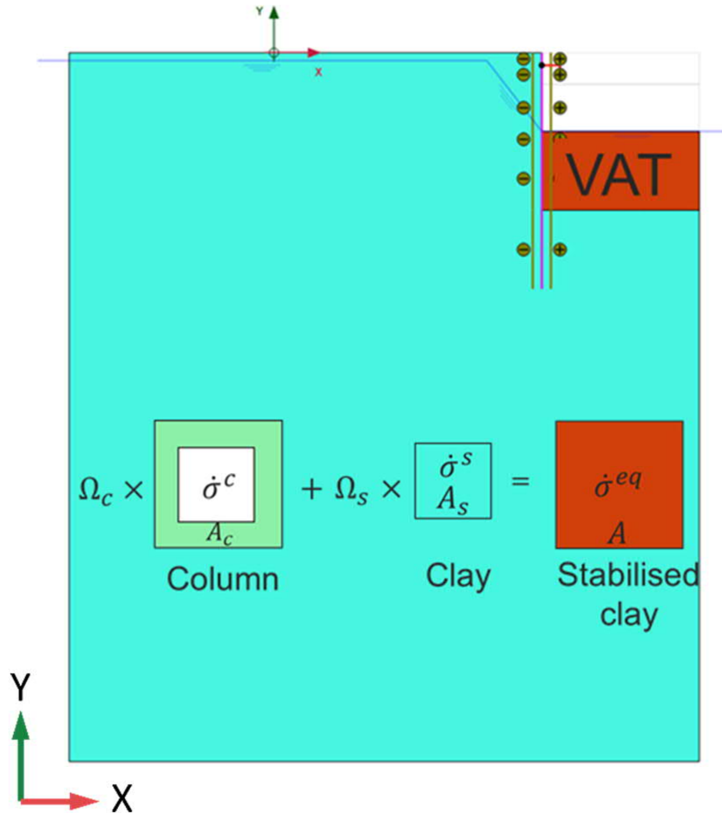


Column : MNhard
Clay : S-CLAY1S

Volume averaging technique (VAT)

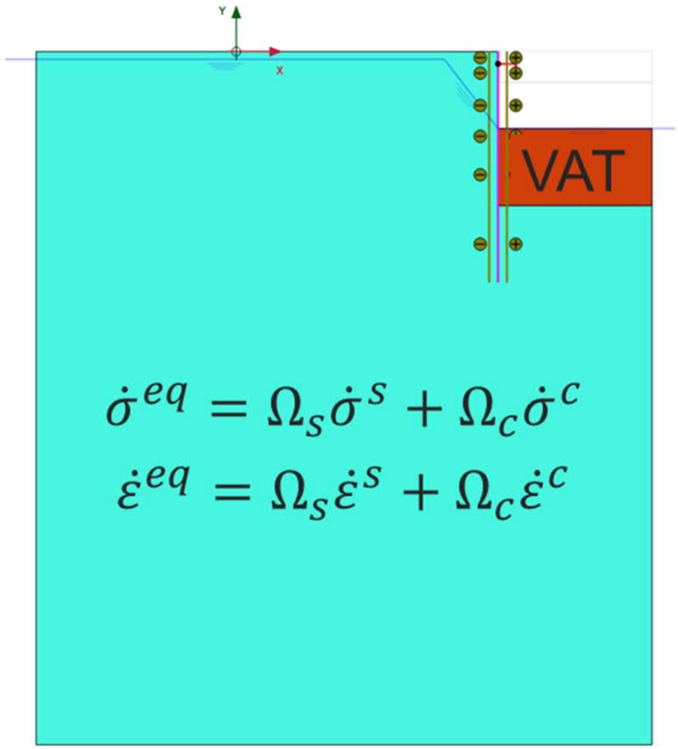


Column : MNhard
Clay : S-CLAY1S



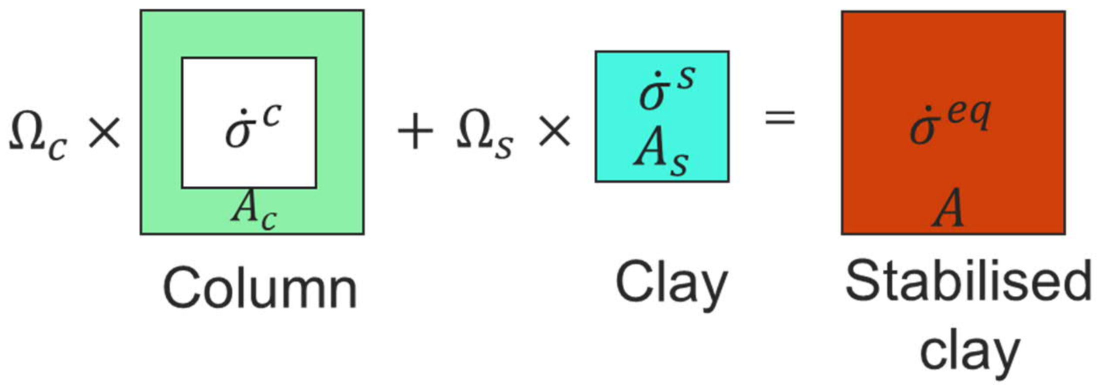
Bozkurt et al. (2024-unpublished)

Volume averaging technique (VAT)



$$\dot{\sigma}^{eq} = \Omega_s \dot{\sigma}^{s} + \Omega_c \dot{\sigma}^{c}$$

$$\dot{\varepsilon}^{eq} = \Omega_s \dot{\varepsilon}^{s} + \Omega_c \dot{\varepsilon}^{c}$$



Column fraction : $\Omega_c = \frac{A_c}{A}$

Soil fraction : $\Omega_s = 1 - \Omega_c$

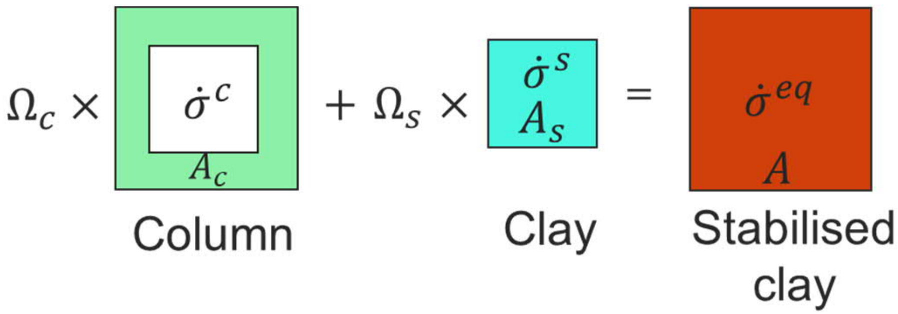
Equivalent stiffness matrix

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix}_{c,s} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} & D_{16} \\ D_{21} & D_{22} & D_{23} & D_{24} & D_{25} & D_{26} \\ D_{31} & D_{32} & D_{33} & D_{34} & D_{35} & D_{36} \\ D_{41} & D_{42} & D_{43} & D_{44} & D_{36} & D_{46} \\ D_{51} & D_{52} & D_{53} & D_{45} & D_{37} & D_{56} \\ D_{61} & D_{62} & D_{63} & D_{46} & D_{38} & D_{66} \end{bmatrix}_{c,s} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix}_{c,s}$$

$$(\dot{\sigma}^{eq})' = \Omega_s(\dot{\sigma}^{s})' + \Omega_c(\dot{\sigma}^{c})'$$

$$\boldsymbol{D^{eq} = \Omega_s D^s S_1^s + \Omega_c D^c S_1^c}$$

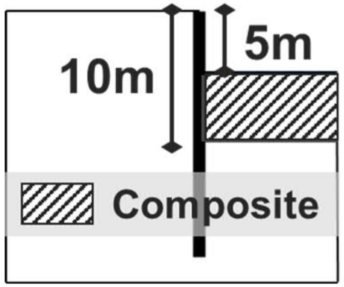
$$S_1^{s,c} = f\,(\Omega_s/\Omega_c, D^s, D^c)$$

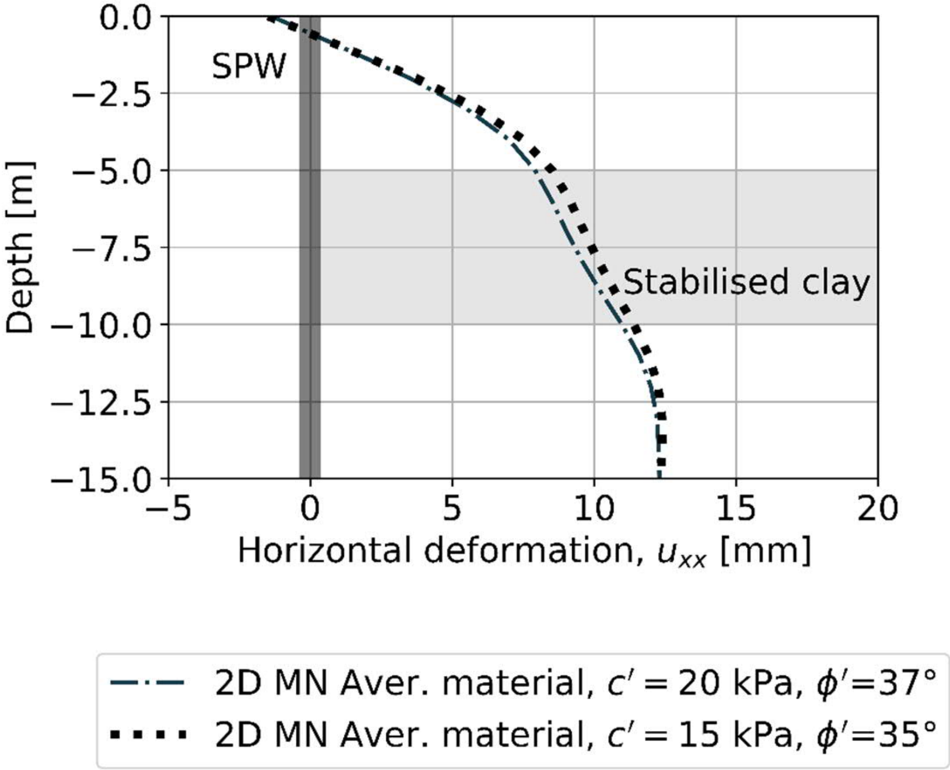


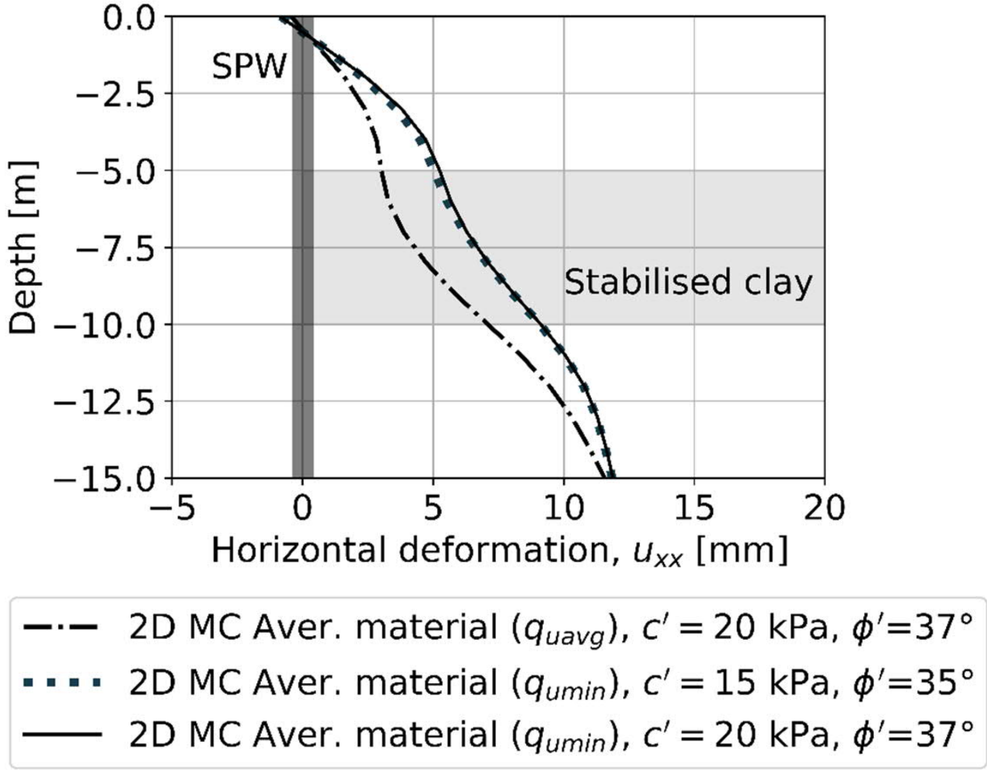
$$\Omega_c = \frac{A_c}{A}$$

$$\Omega_s = 1 - \Omega_c$$

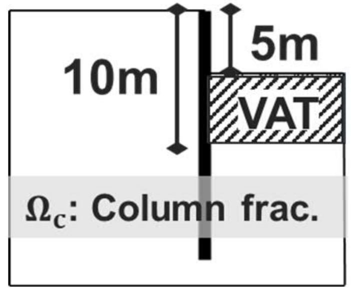
Nonlinear vs linear soil response

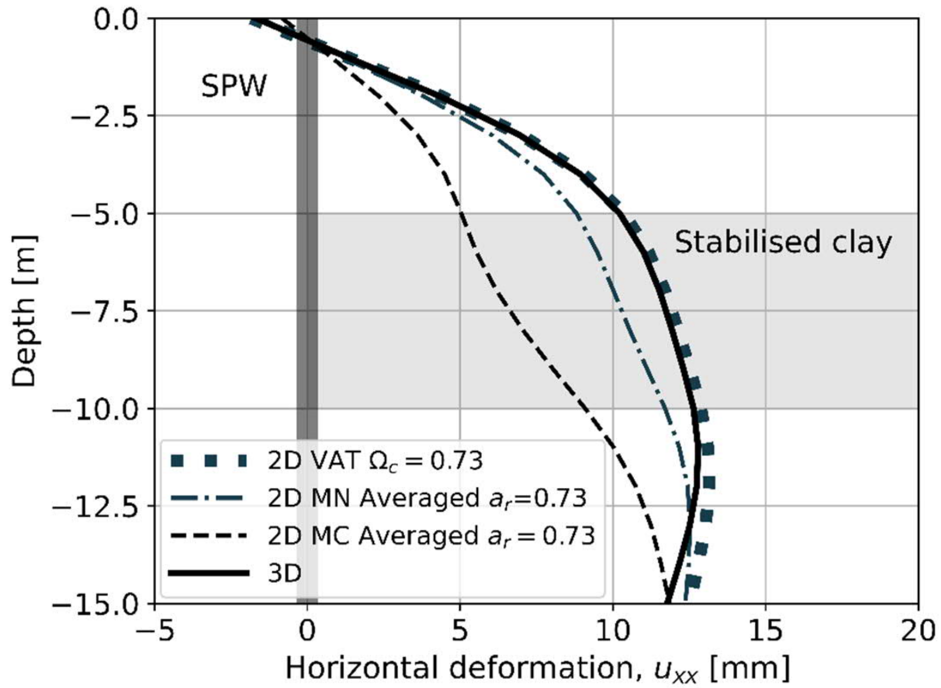


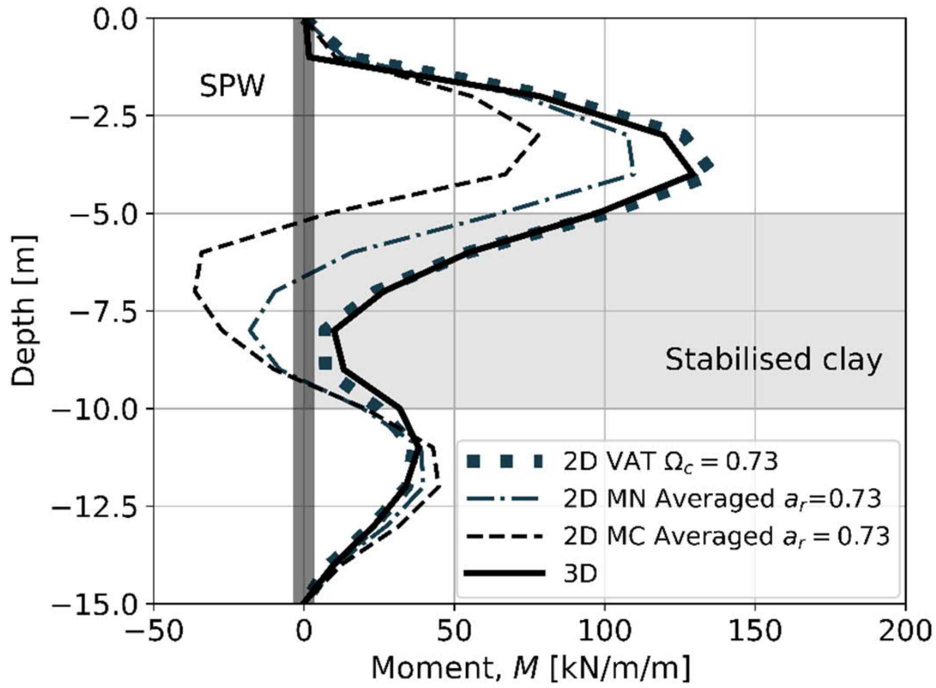




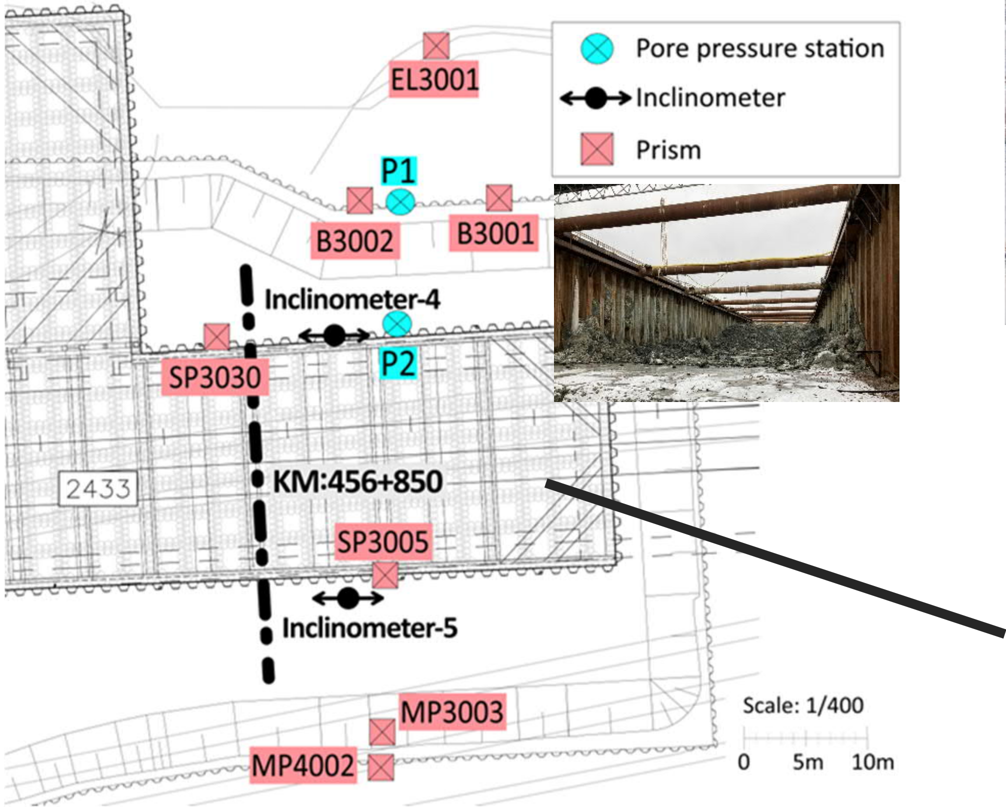
Comparison of alternative methods

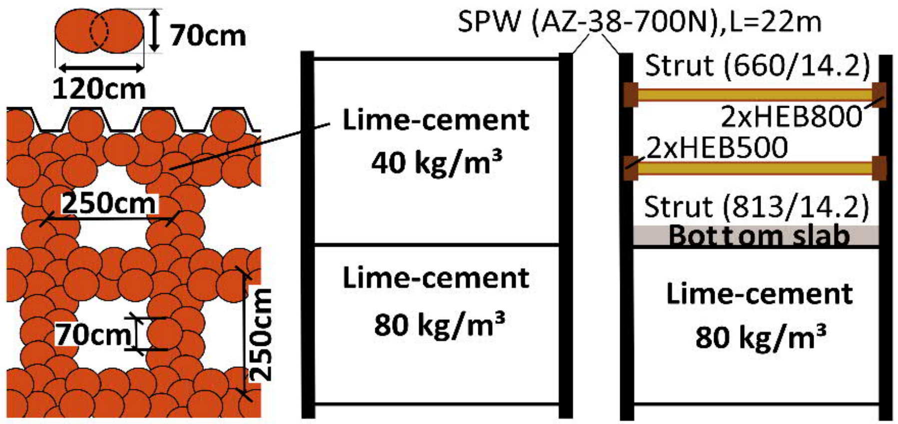






Site location





Conclusions

- One-dimensional compression tests are inadequate for modelling stabilised clay.
- Accurate estimation of the overall response of stabilised excavations requires considering the responses of both *in situ* clay and stabilised clay.
- Using soil models that employ hardening/softening plasticity and stress-dependent stiffness ensures a realistic representation of stabilised clay behaviour.
- The VAT has proven to be an efficient tool for the numerical analysis of stabilised deep excavations.

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